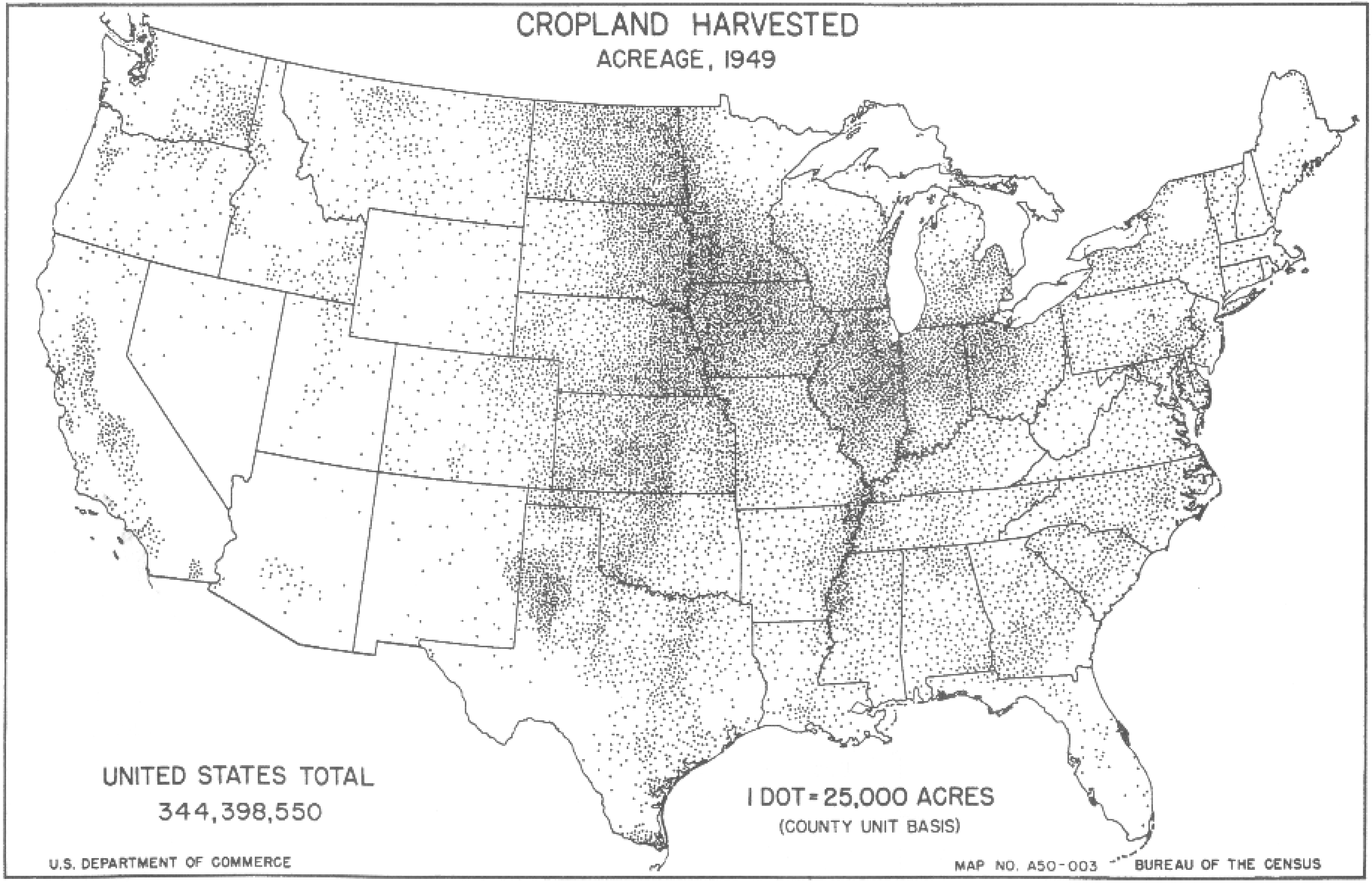
" Dotting The Dot Map, Revisited "



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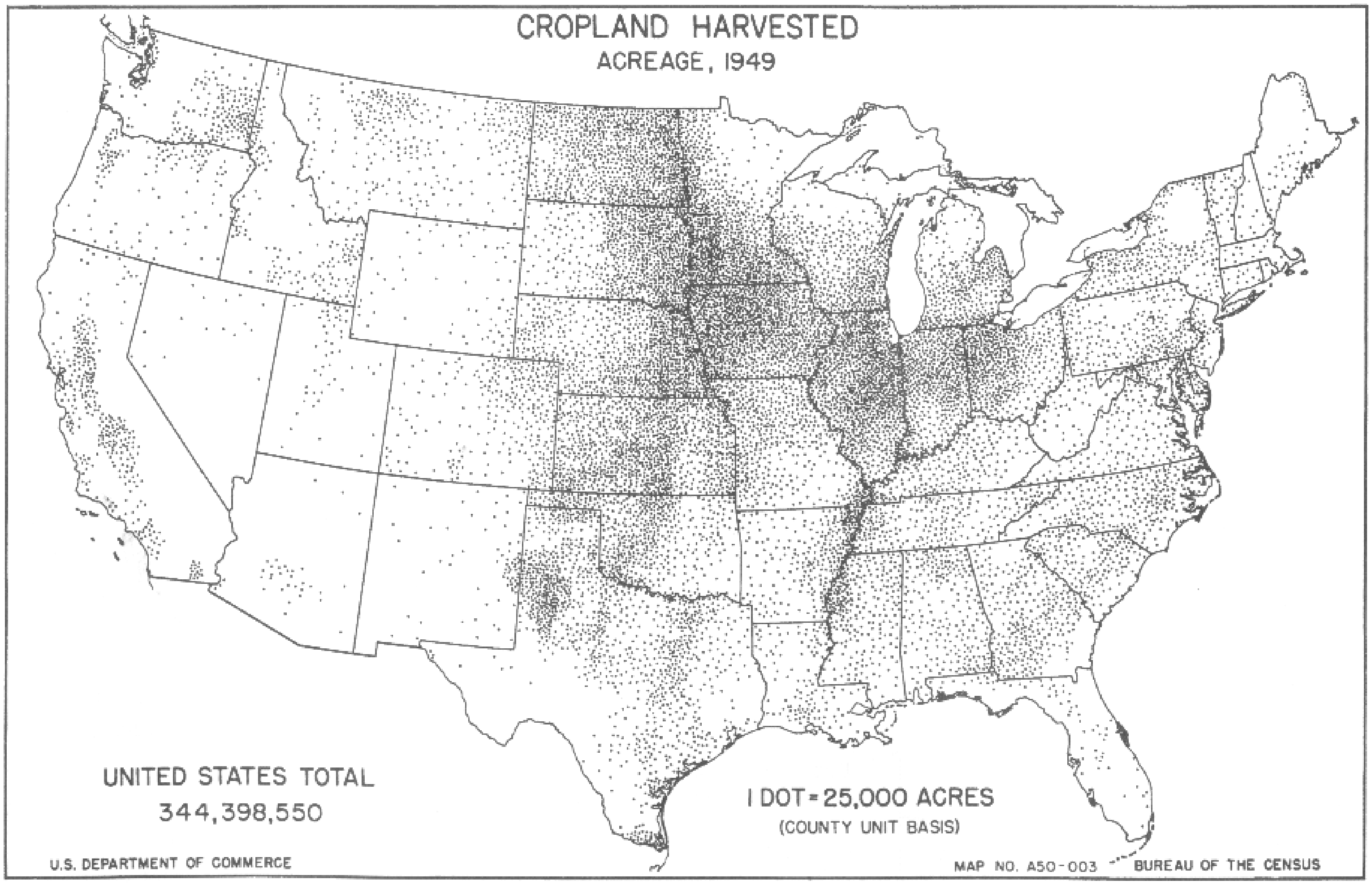
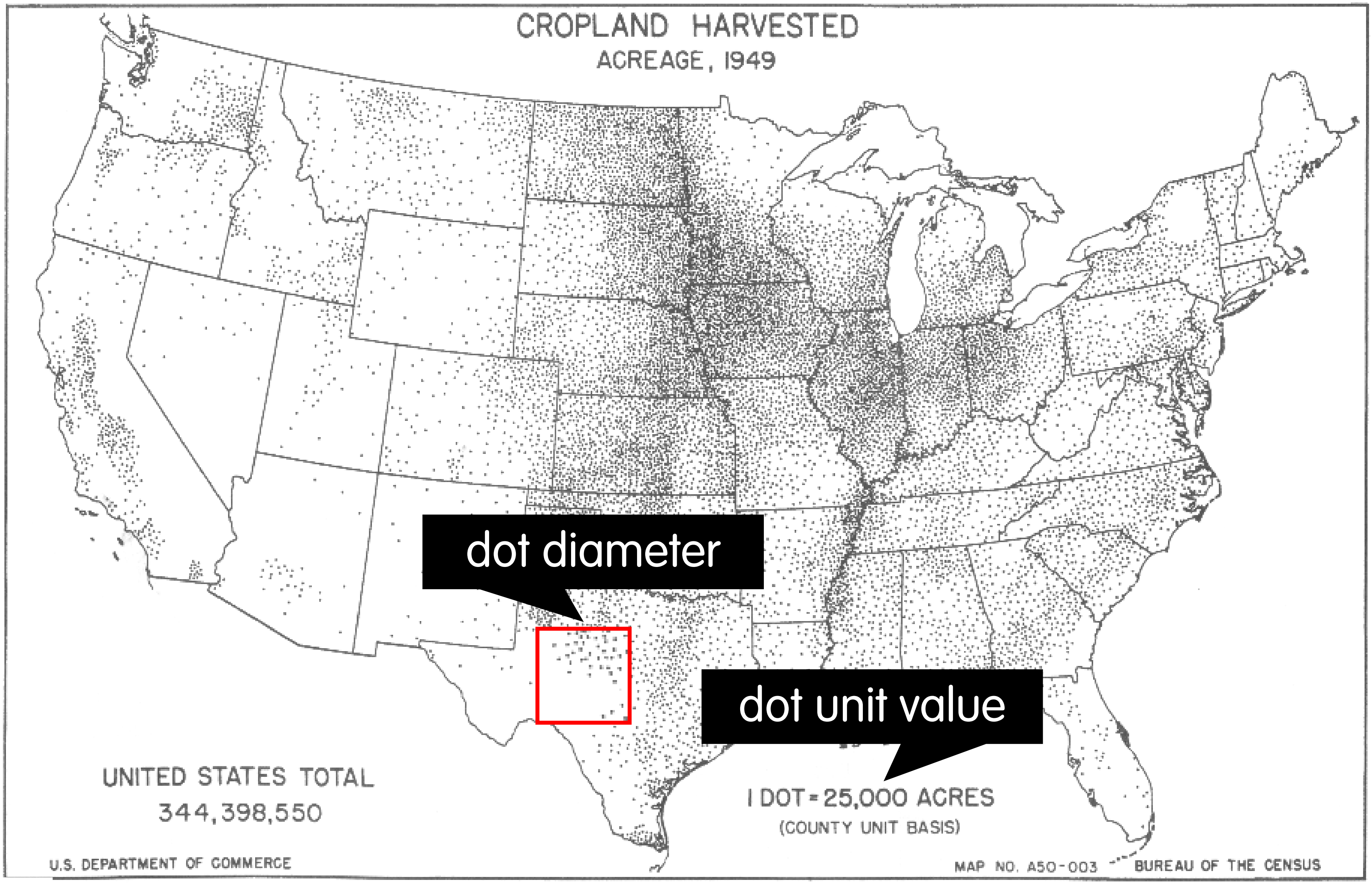
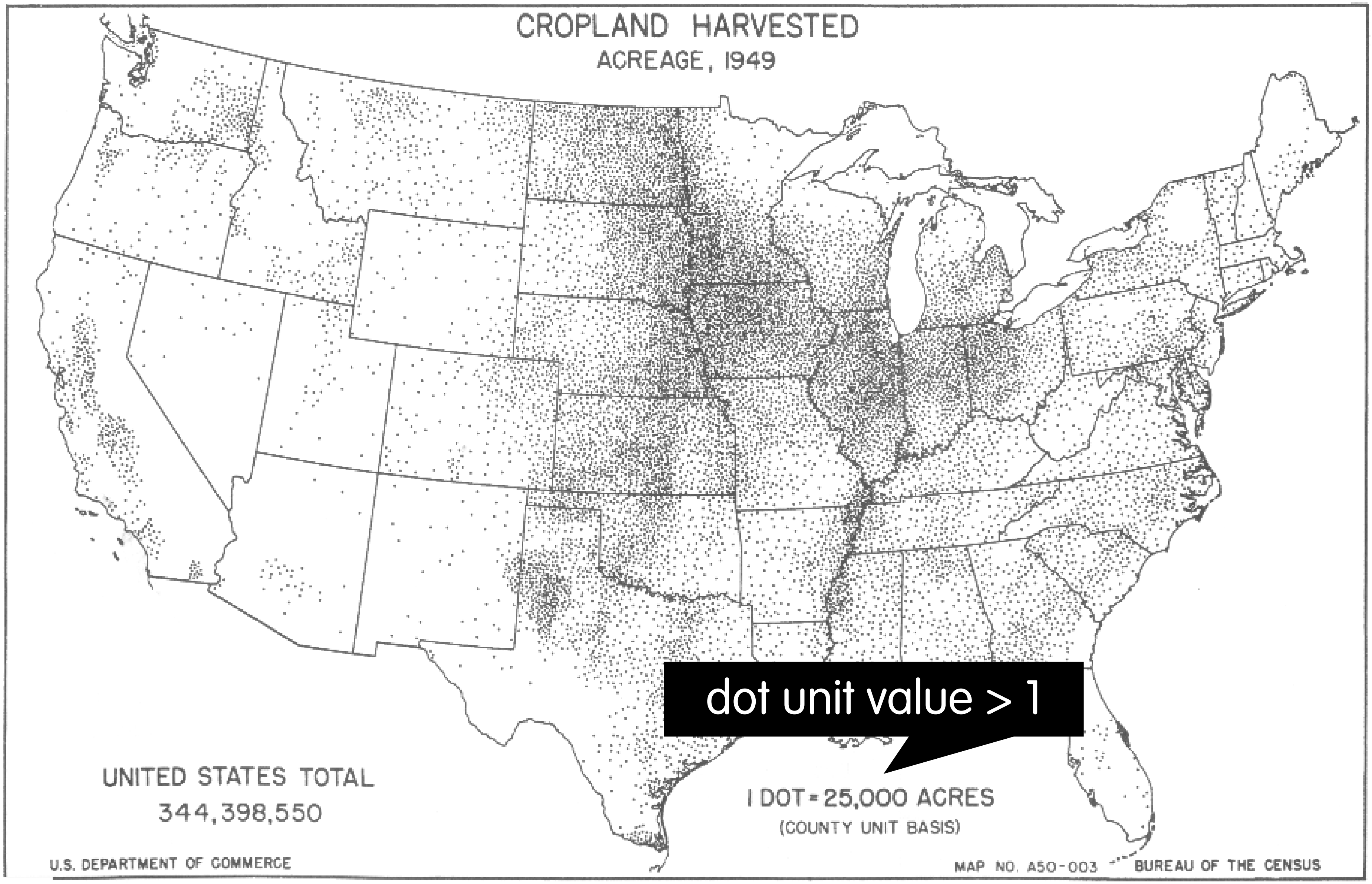
Dot maps show the geographic distribution of features in an area by placing dots representing a certain quantity of features where the features are most likely to occur.



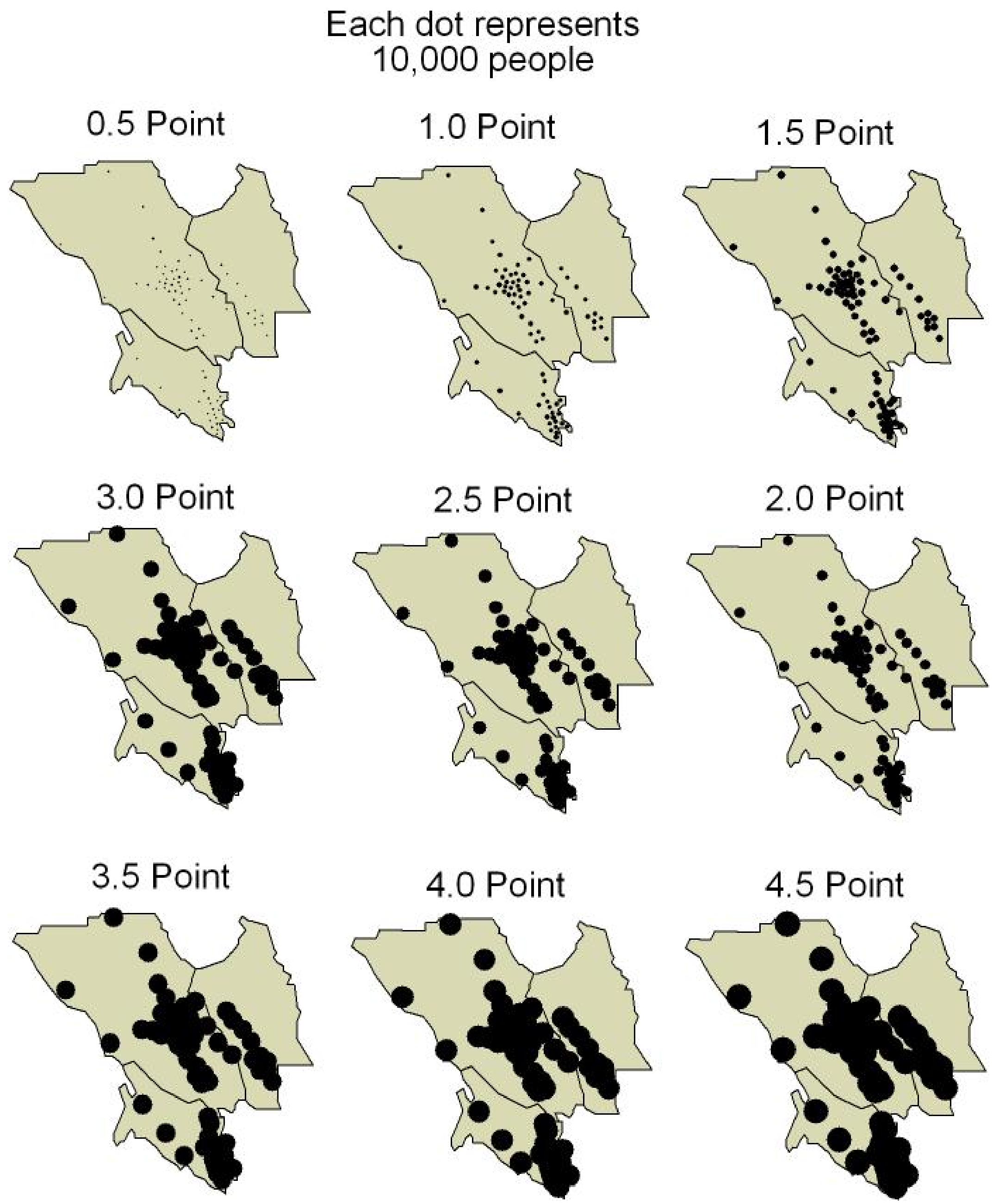
When making a dot map, we first select a dot diameter and then determine the dot unit value.



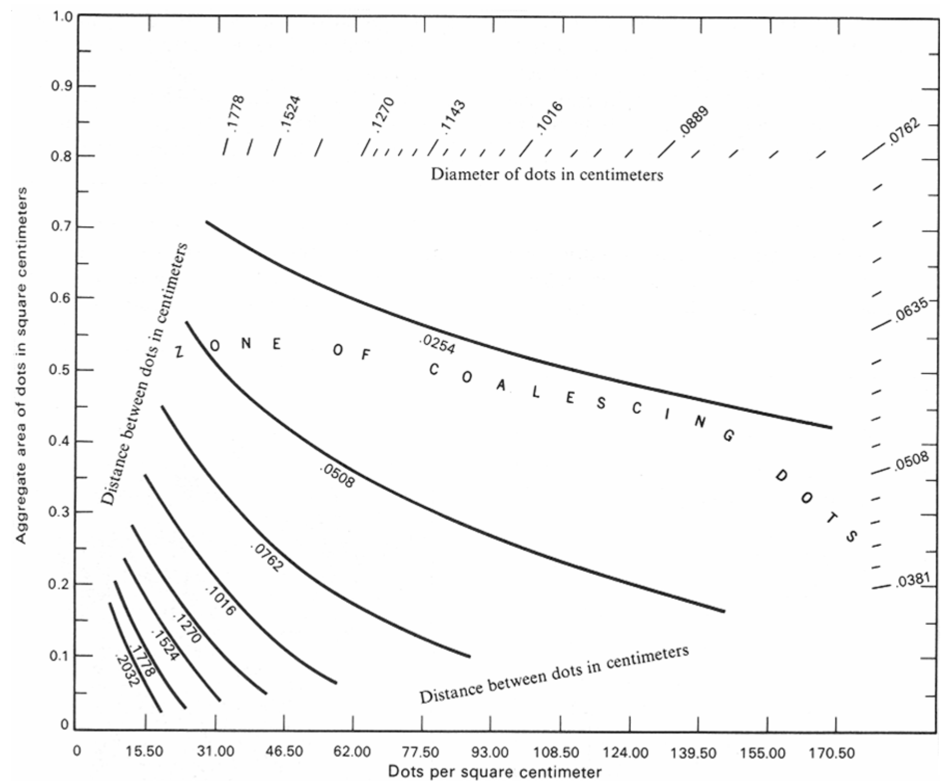
The dot unit value is always greater than one.



Selecting the dot diameter is a subjective decision.



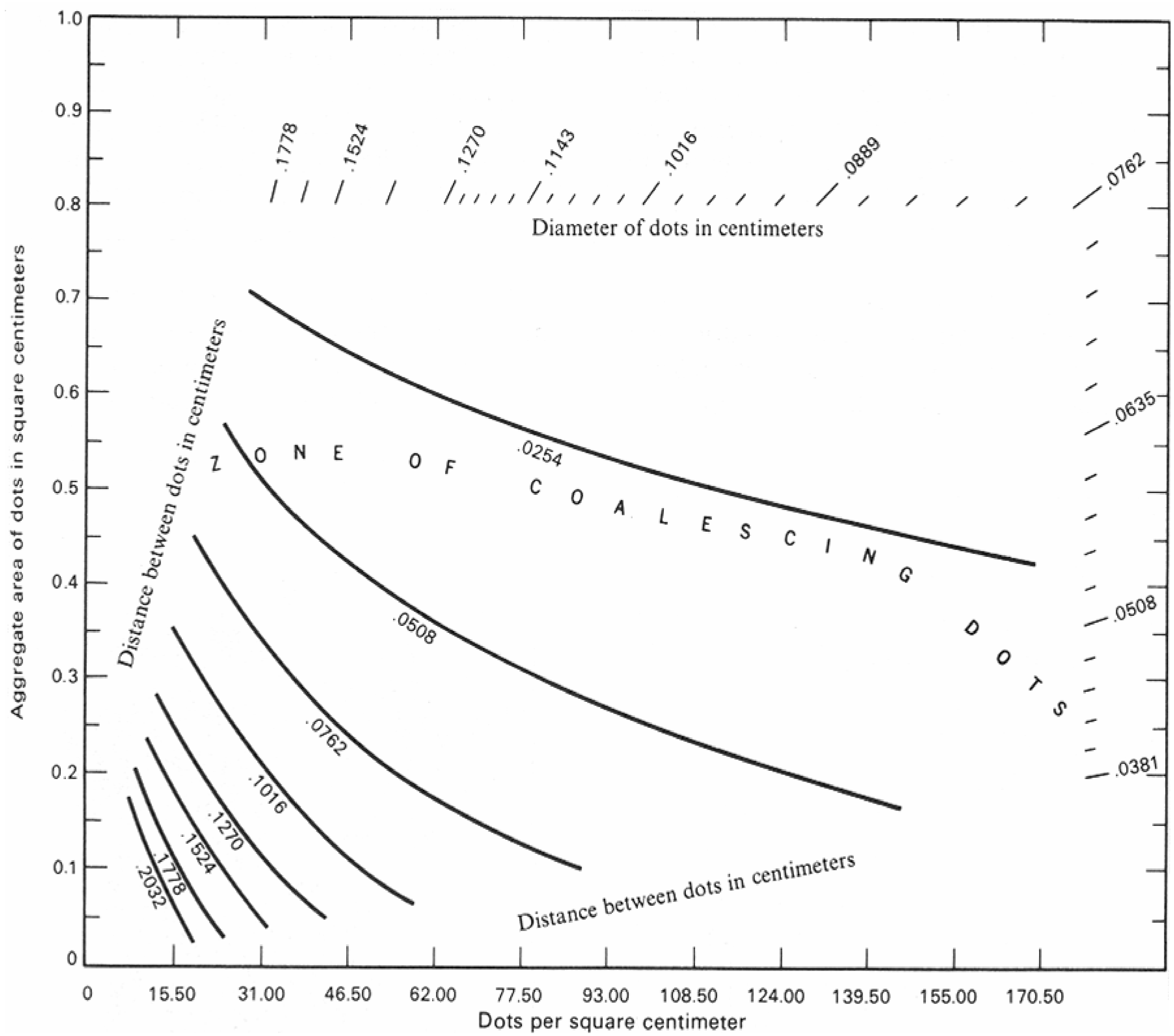
Selecting the dot unit value is done by trial and error or using the Mackay nomograph.



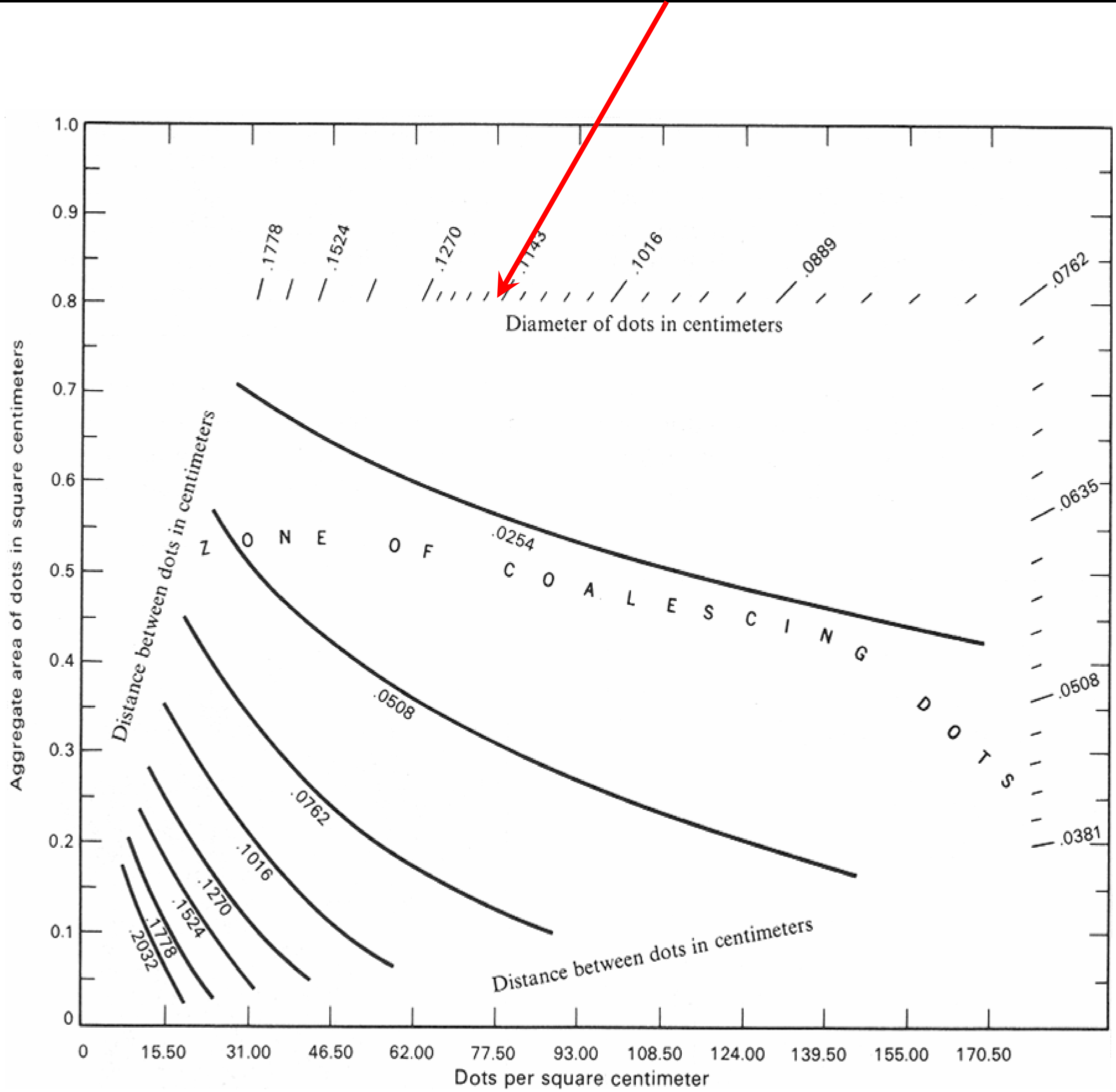
J. Ross Mackay, 1949

nom·o·graph –noun a graph, usually containing three parallel scales graduated for different variables, designed to solve an equation; also called an alignment graph or calculation graph.

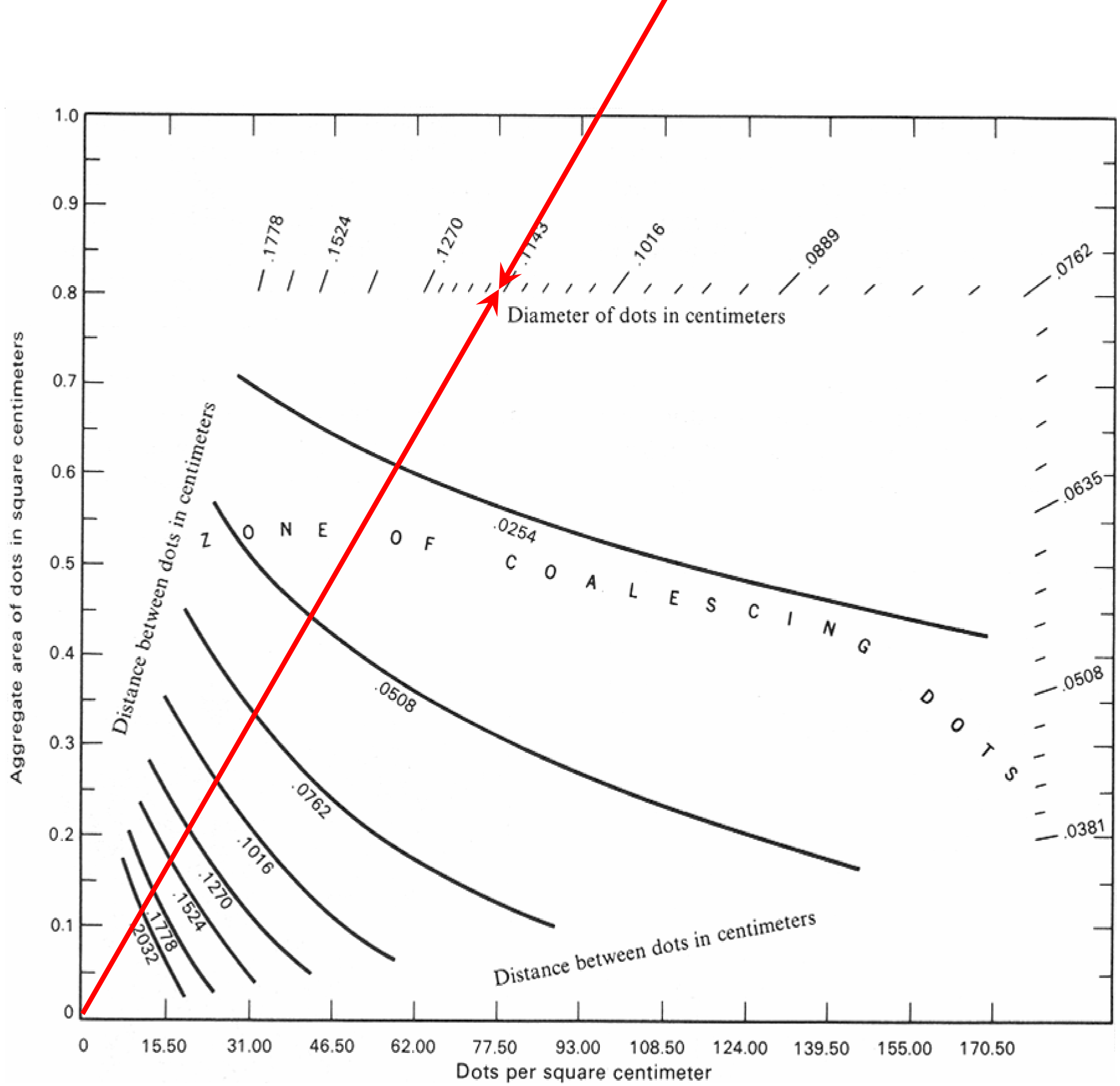
Using the Mackay nomograph to find the dot unit value…



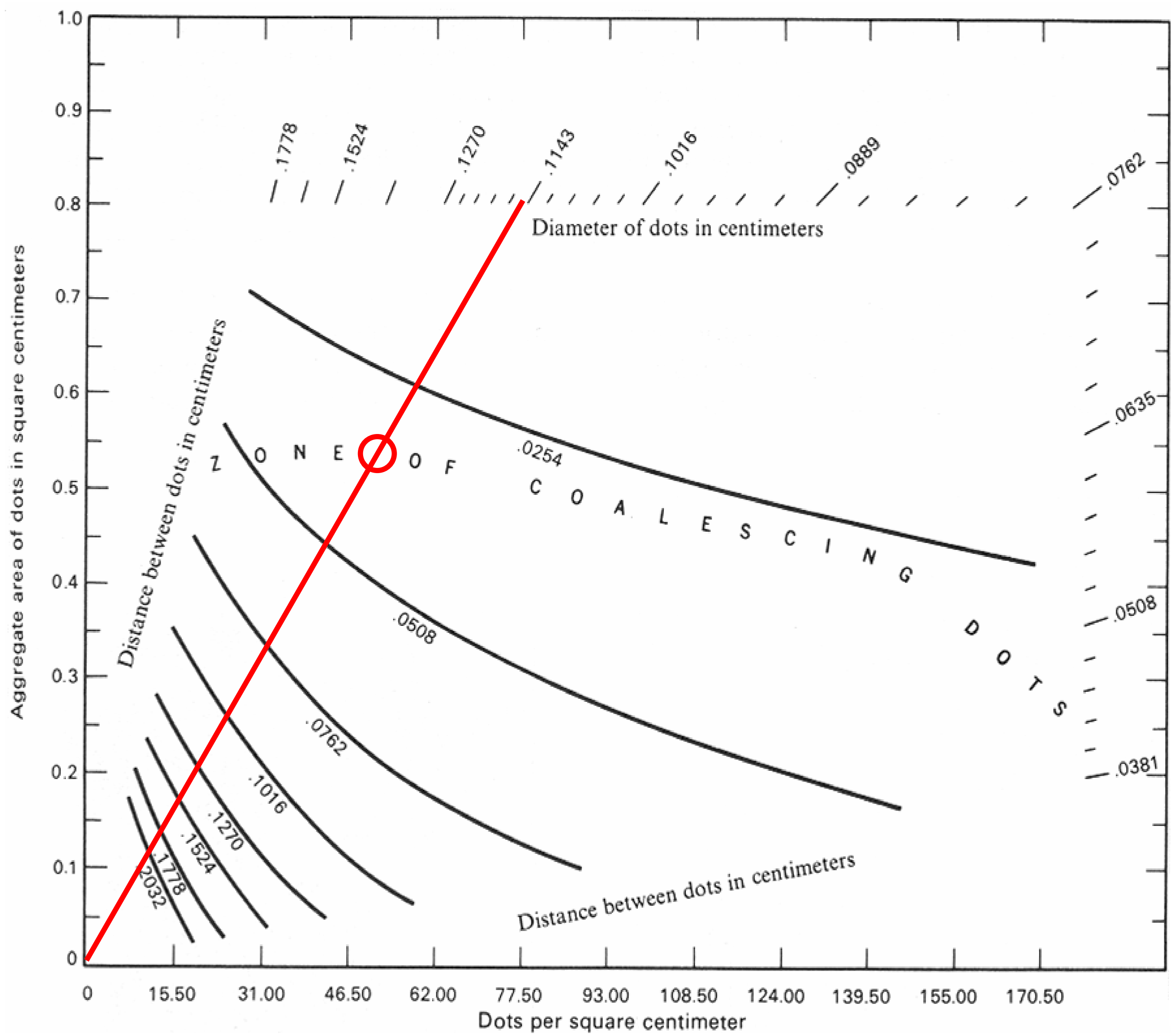
Mark the selected dot diameter…



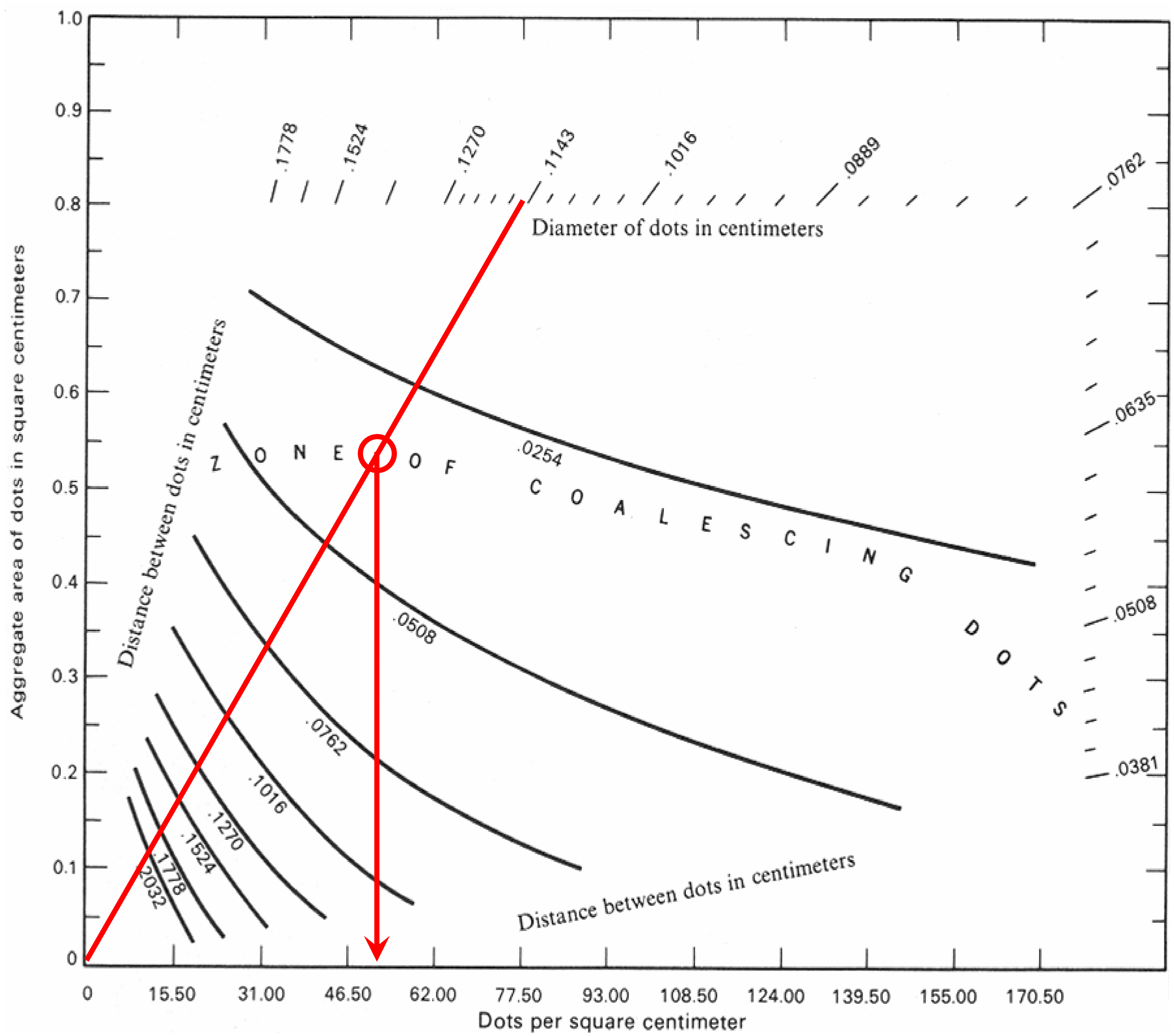
Draw a line from the origin to your mark…



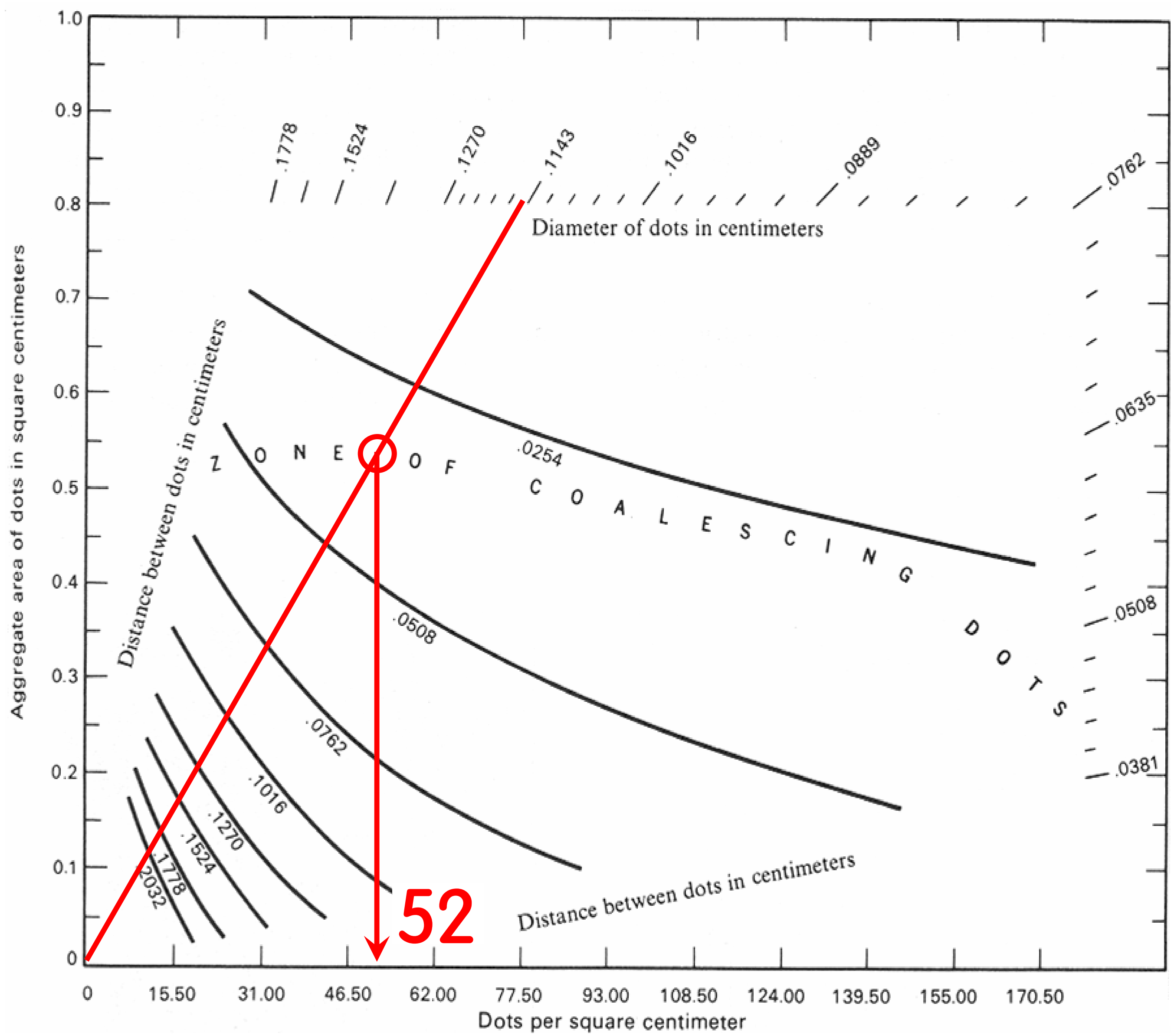
Find where the line crosses the zone of coalescing dots…



Draw a vertical line down to the x-axis…

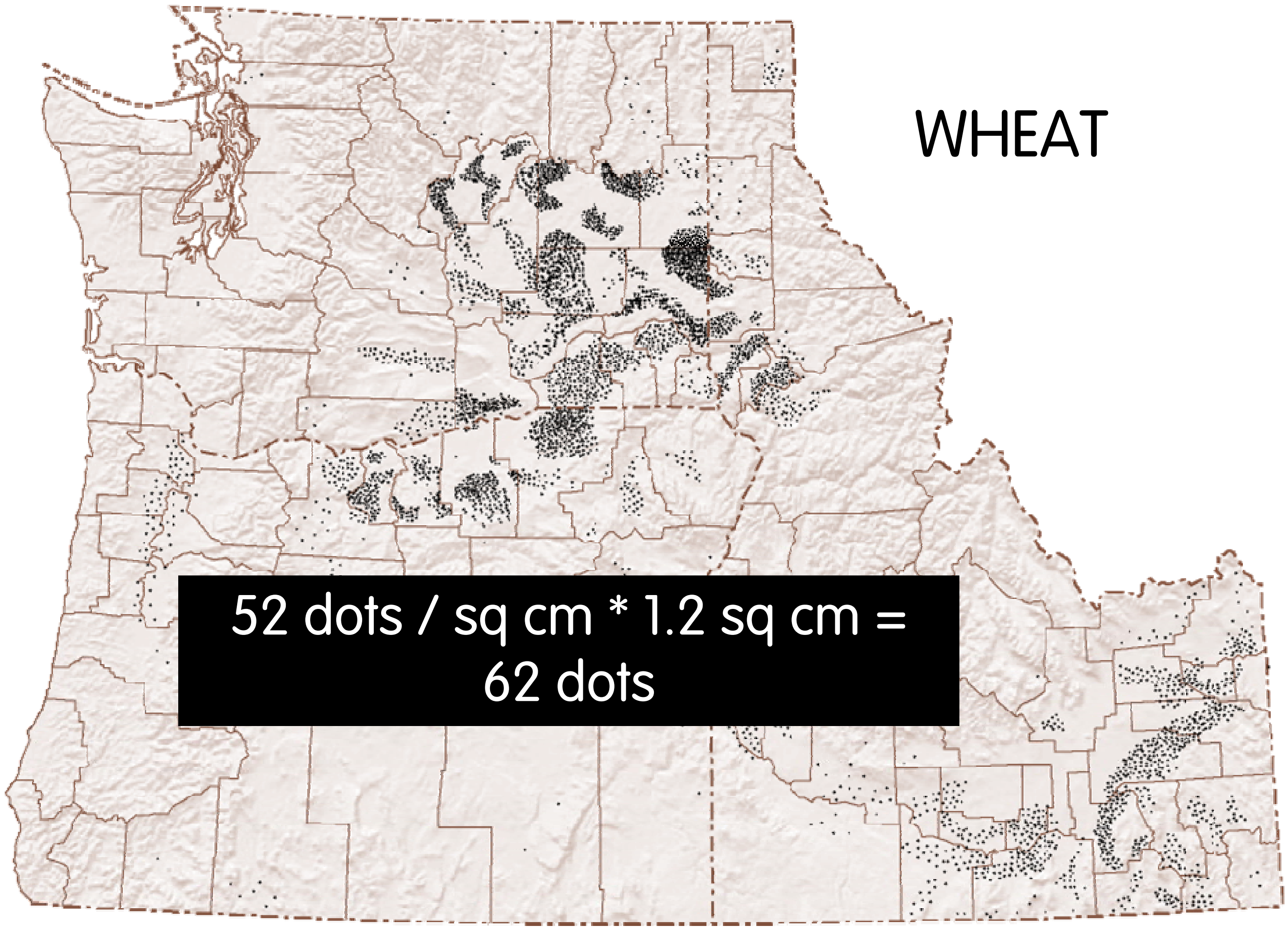


Note the number of dots per square centimeter.

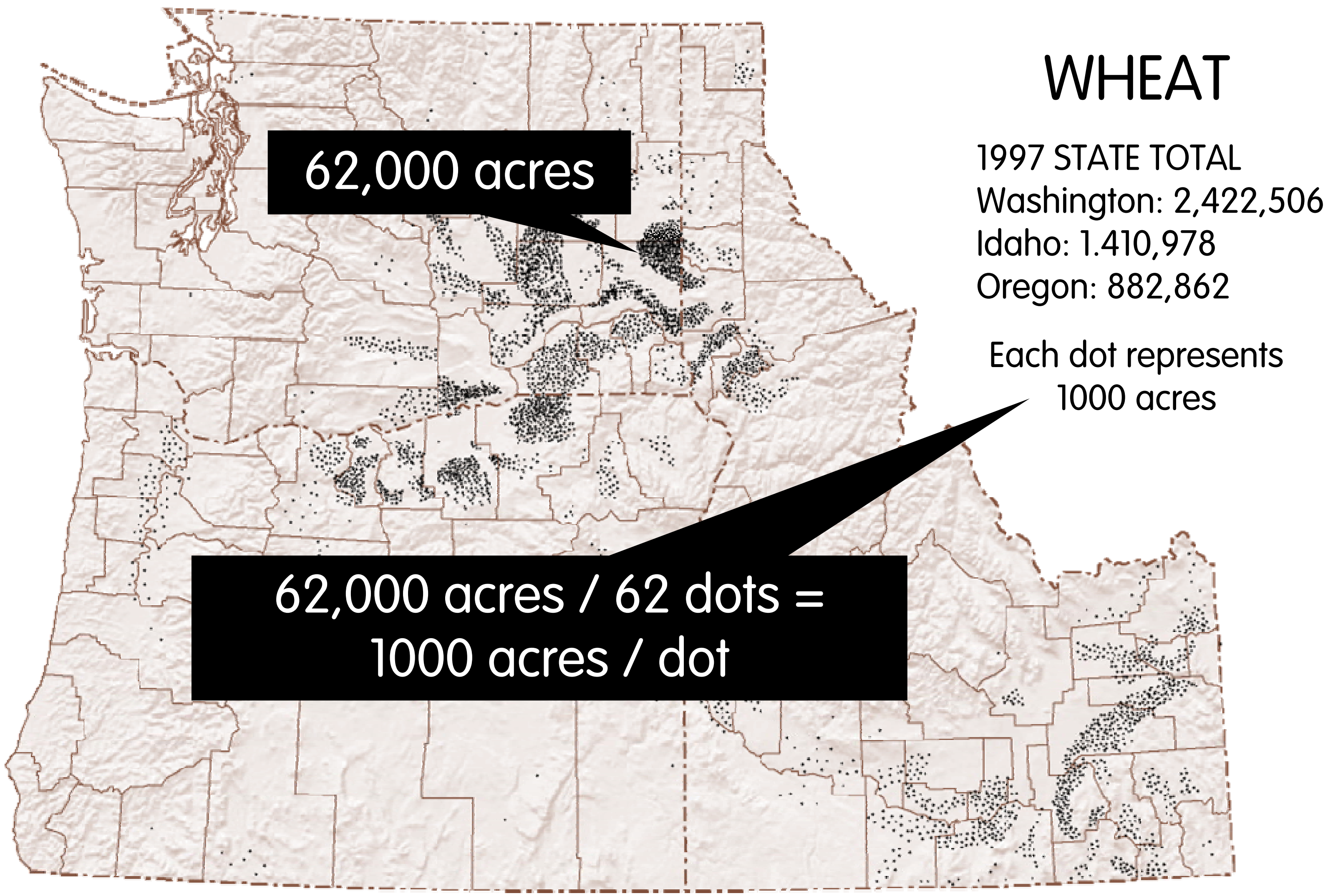


Then, find the map area of the region that will have the highest density of dots…

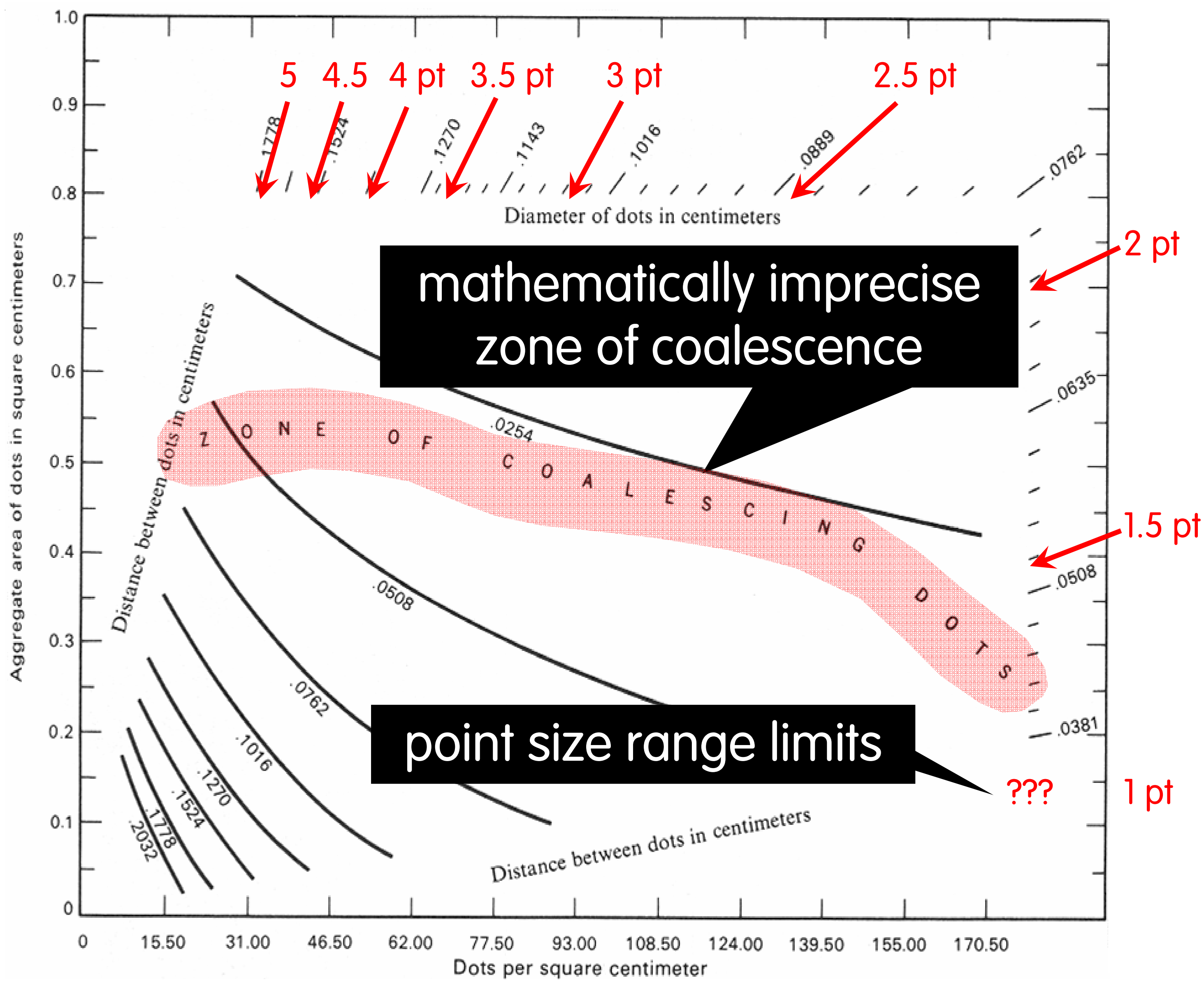
Multiply the map area by the number of dots per sq. cm. from the nomograph to find the number of dots to place in that area…



Divide the quantity in the densest area by the number of dots to find the dot unit value…

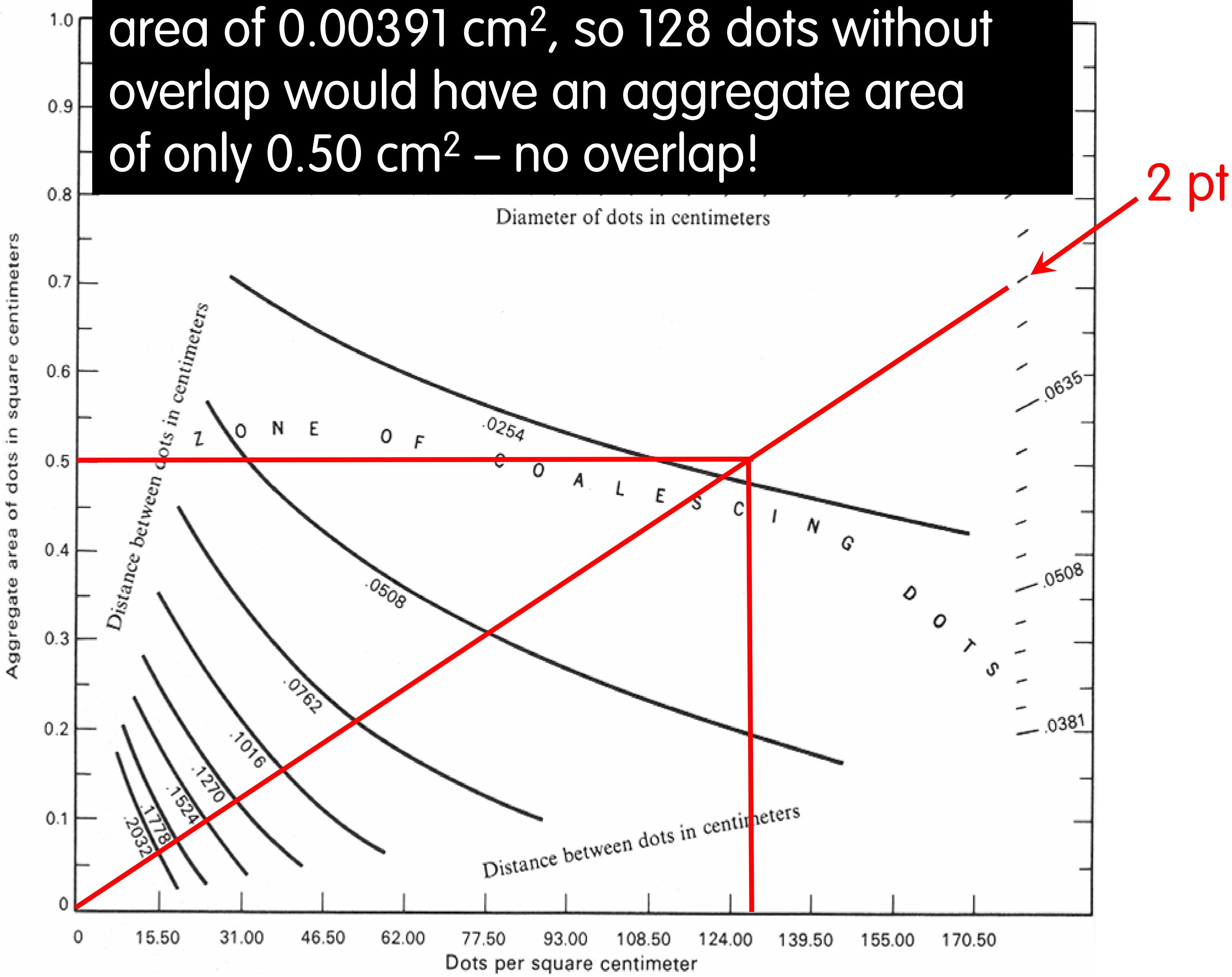


Issues with the *Mackay* nomograph…



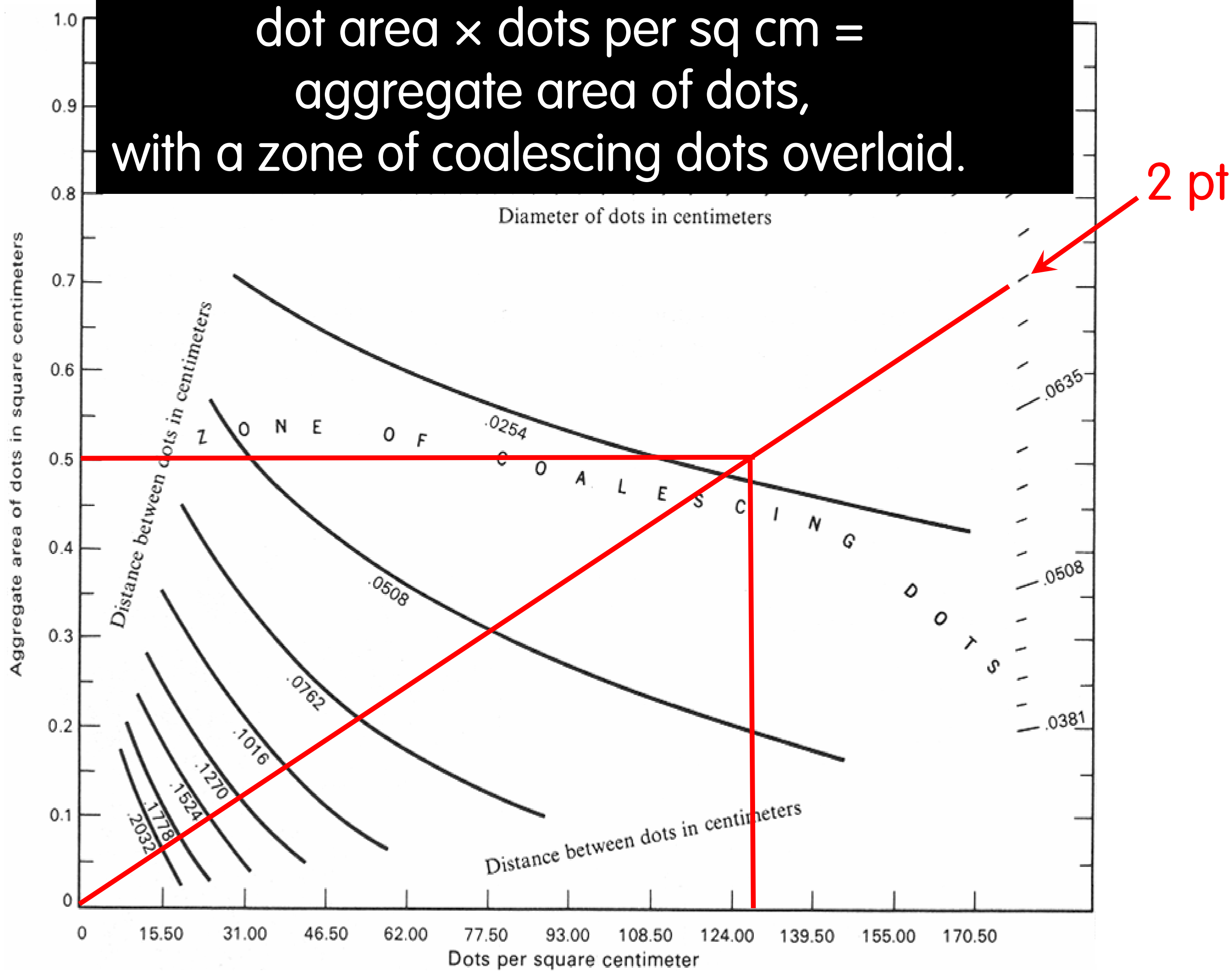
Let's look carefully at the nomograph…

A 2 pt dot with diameter 0.0706 cm has an area of 0.00391 cm², so 128 dots without overlap would have an aggregate area of only 0.50 cm² – no overlap!

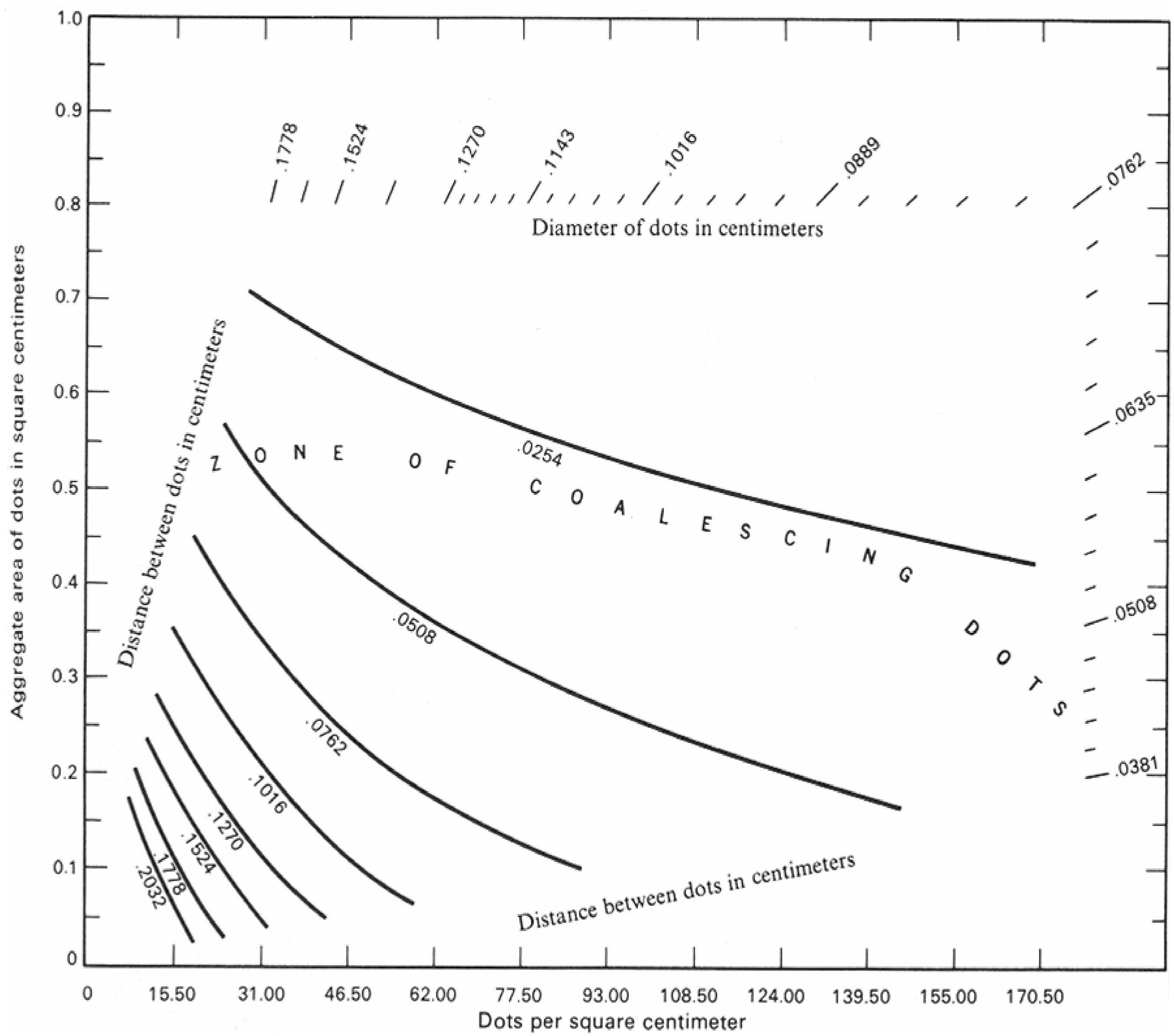


Let's look carefully at the nomograph…

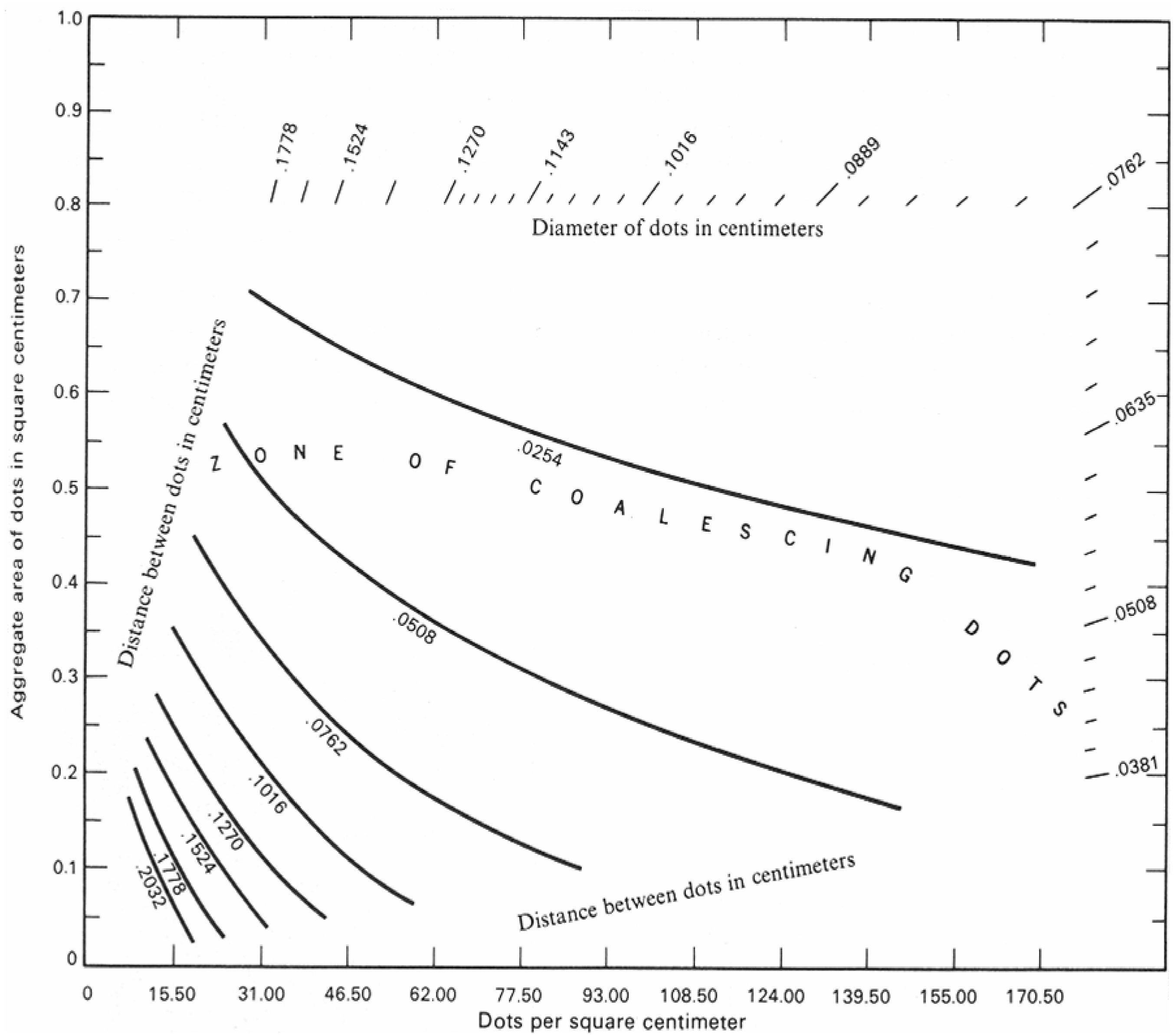
The MacKay nomograph is a graph of :
dot area × dots per sq cm =
aggregate area of dots,
with a zone of coalescing dots overlaid.



We need a theoretically sound mathematical basis for dot coalescence.



Modeling dot coalescence using the Unification Equation from probability theory is the answer!



The Unification Equation…

$$P\left(\bigcup_{i=0}^{n}(E_i)\right)=\sum_{i=0}^{n}P(E_i)-\sum_{i<j}P(E_iE_j)+\sum_{i<j<k}P(E_iE_jE_k)-\ldots+(-1)^{n+1}\sum_{i<j<k\ldots<n}P(E_1E_2E_3\ldots E_n)$$

The Dot Coalescence Model

A 1 cm^2 square has a probability P of 1.0.

- The area of each of dot *(Ei)* in proportion to the area of the square is its probability $p = P(E_i)$.

Since the *n* dots in the square are the same size,

$E_i = E_j = \ldots = E_n$ and $p = P(Ei) = P(Ej) = \ldots = P(En)$

The Unification Equation…

$P\left(\bigcup_{i=0}^{n}(p)\right)$ is the aggregate dot area in cm^2

$$P\left(\bigcup_{i=0}^{n}(p)\right) = \sum_{i=0}^{n} p - \sum_{i<j} p^2 + \sum_{i<j<k} p^3 - \ldots + (-1)^{n+1} \sum_{i<j<k\ldots<n} p^n$$

Summations are numbers of dot combinations

$$\sum_{i=0}^{n} p = np$$

The number of dot combinations in each summation are found by:

$$\frac{n!}{k!(n-k)!}$$

So…

$$P\left(\bigcup_{i=0}^{n}(p)\right) = np - \frac{n!}{2!\,(n-2)!}\,p^{2} + \frac{n!}{3!\,(n-3)!}\,p^{3} - \ldots + (-1)^{n+1}\,\frac{n!}{k!\,(n-k)!}\,p^{k}$$

There is a term in the series for each dot added, but I truncated the series at $k = 10$ without affecting the results for up to:

$$P\left(\bigcup_{i=0}^{n}(p)\right) = 0.95$$

with n up to $1{,}000$ points.

So…

$$P\left(\bigcup_{i=0}^{n}(p)\right) = np - \frac{n!}{2!\left(n-2\right)!}\,p^{2} + \frac{n!}{3!\left(n-3\right)!}\,p^{3} - \ldots + \left(-1\right)^{10}\frac{n!}{10!\left(n-10\right)!}\,p^{10}$$

Or doing the factorials…

$$P\left(\bigcup_{i=0}^{n}(p)\right) = np - \frac{n(n-1)}{2}\,p^{2} +$$

$$\frac{n(n-1)(n-2)}{6}\,p^{3} - \ldots$$

$$+\left(-1\right)^{10}\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-8)(n-9)(n-10)}{3{,}628{,}800}\,p^{10}$$

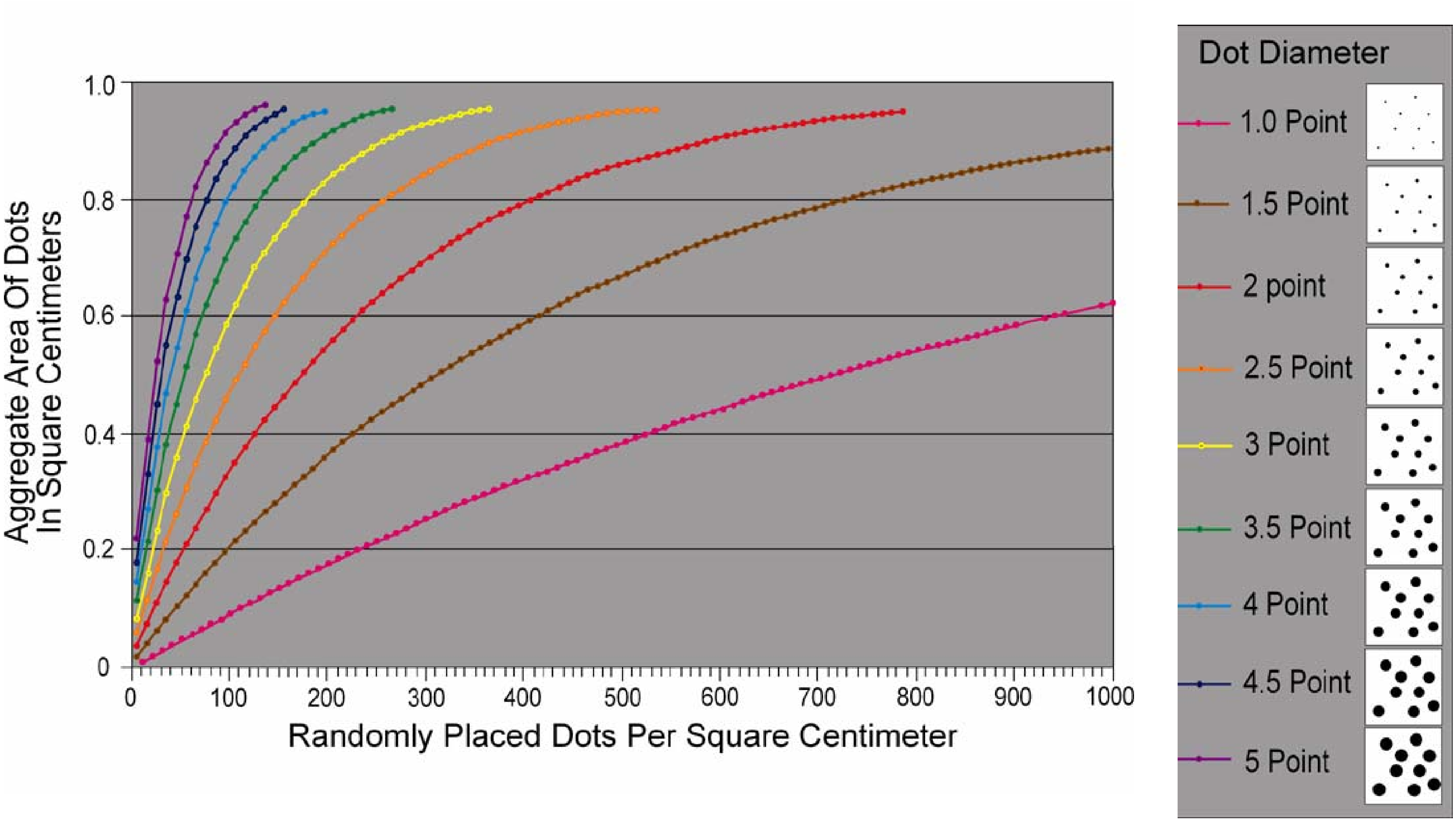
Using the equation with ESRI dots…

ESRI dots are in 0.5 Postscript point increments from 0.5 to 11 points, although sizes smaller than 1 or larger than 5 points would not normally be used for dot mapping.

Given that 1 Postscript point = 0.353 mm, the following table gives values of *p* (dot areas) for this dot size range.

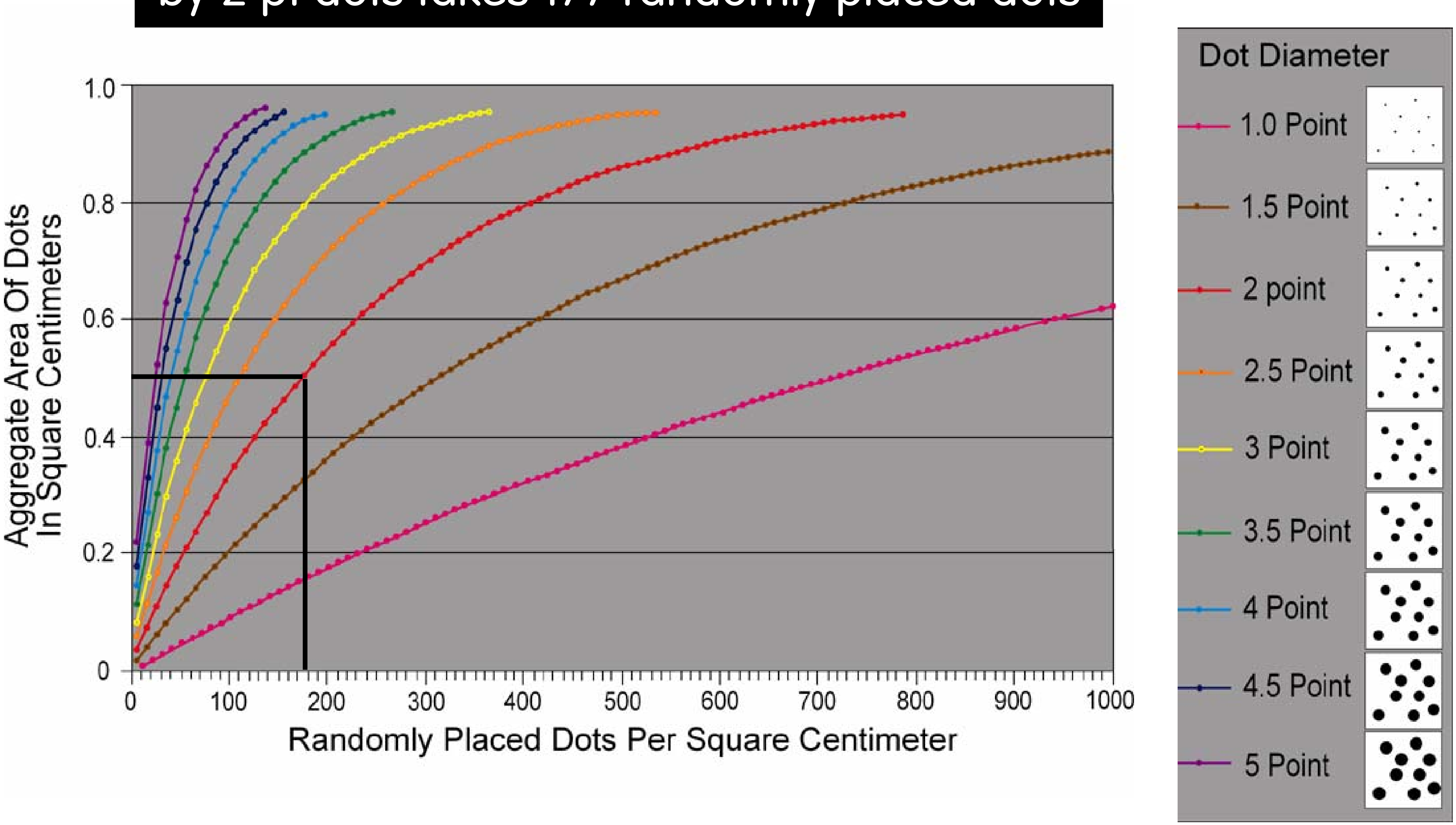
Point Size	p (cm^2)	Point Size	p (cm^2)
1.0	0.000978	3.5	0.011988
1.5	0.002202	4.0	0.015658
2.0	0.003914	4.5	0.011988
2.5	0.006116	5.0	0.024466
3.0	0.008808		

Running the equation with ESRI dots in 10 dot increments gave the results plotted on this graph.

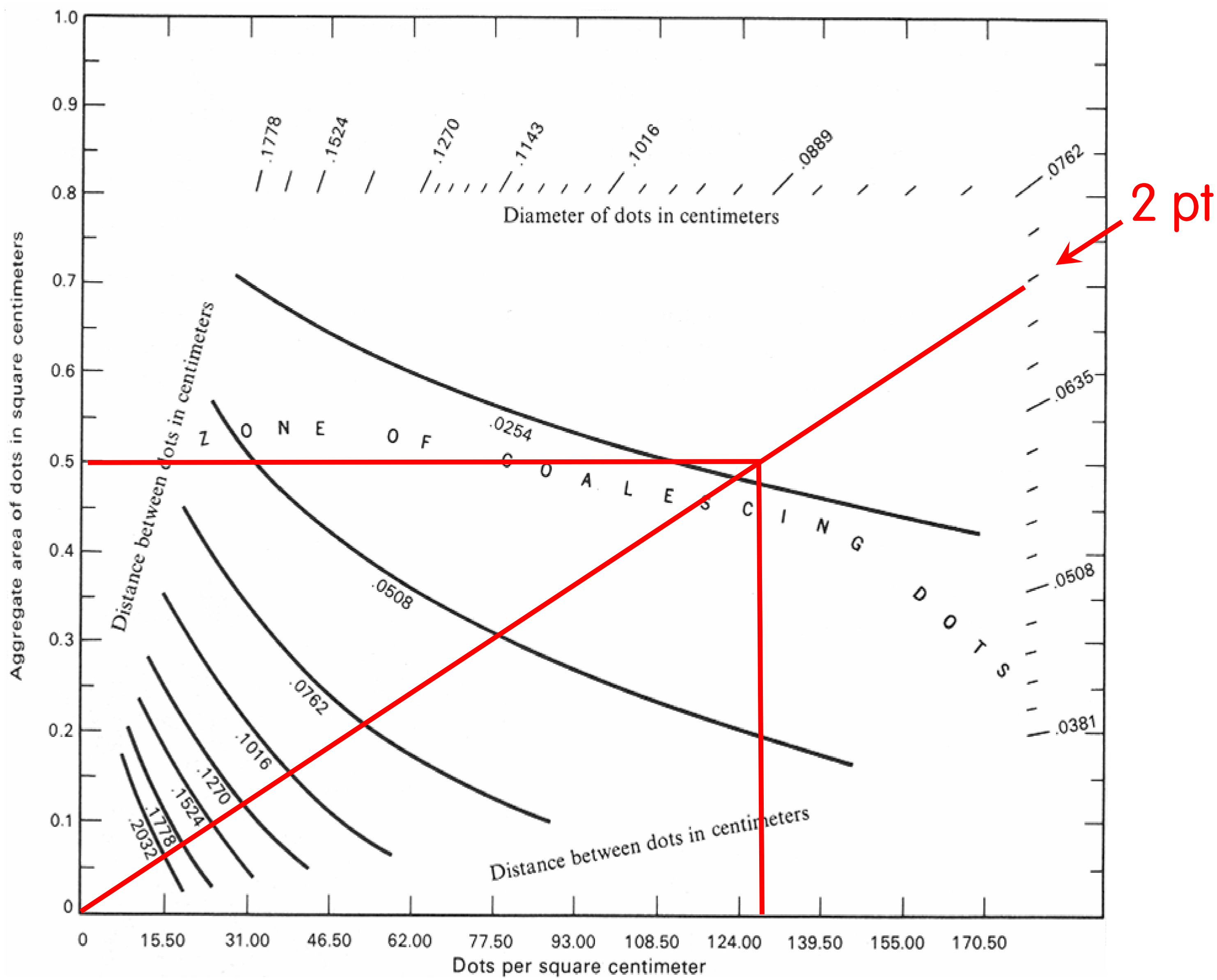


Using the graph…

Example: Having half the 1 cm² area covered by 2 pt dots takes 177 randomly placed dots



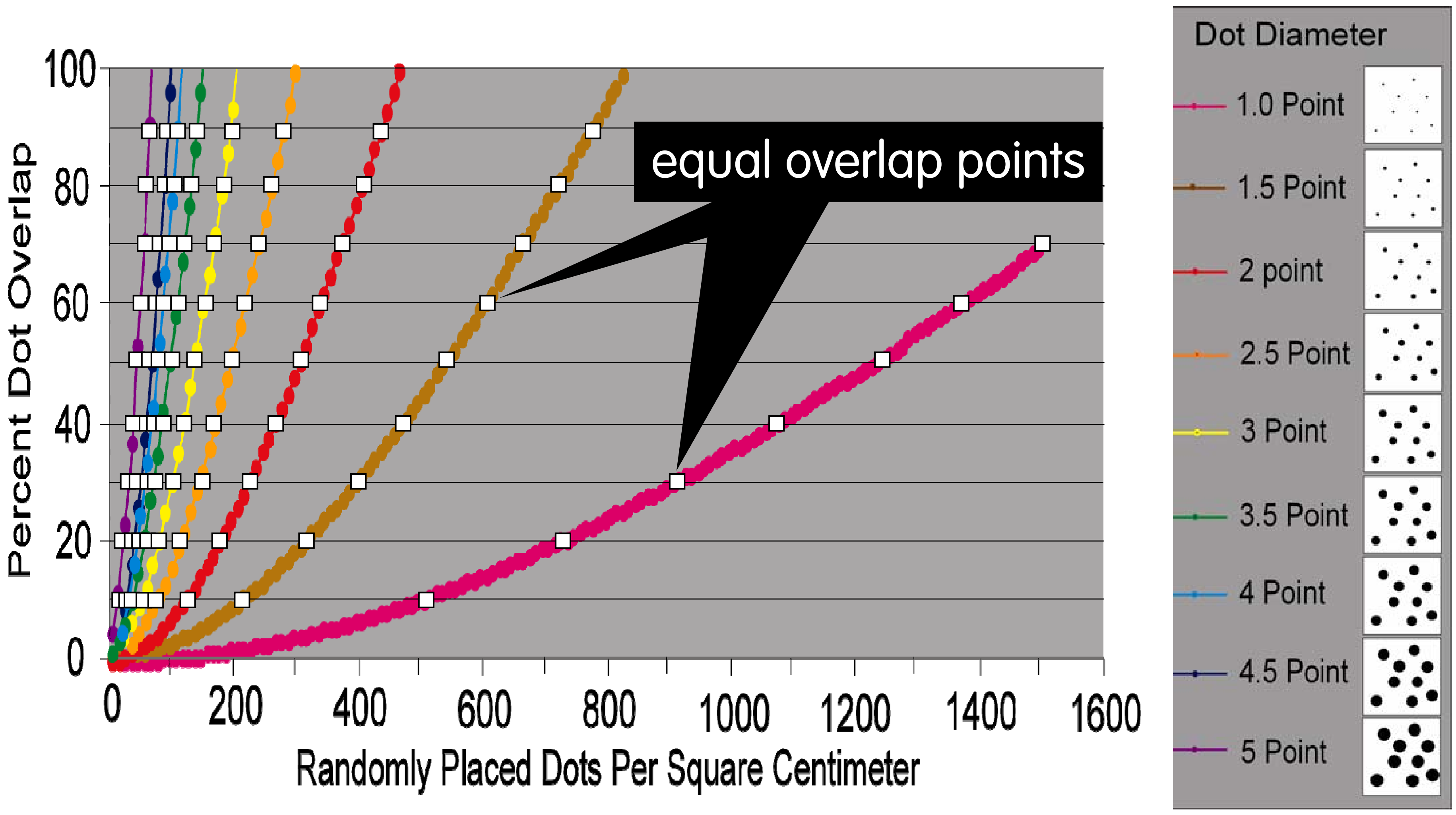
This compares with 128 dots from the MacKay nomograph.



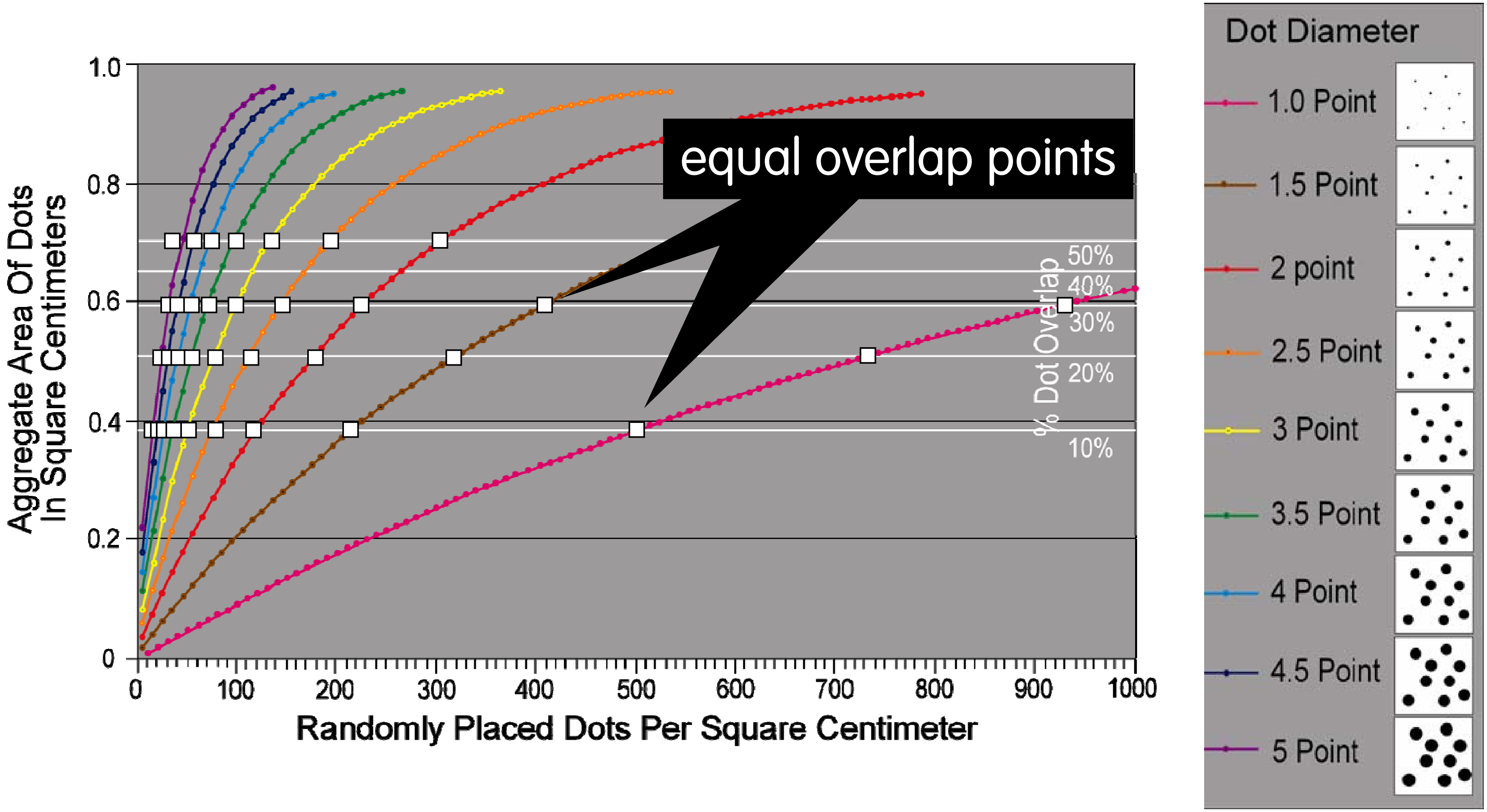
Amount of dot overlap is another measure of coalescence.

$$\text{Percent Overlap} = \left(np - P\left(\bigcup_{i=0}^{n} (p) \right) \right) \times 100$$

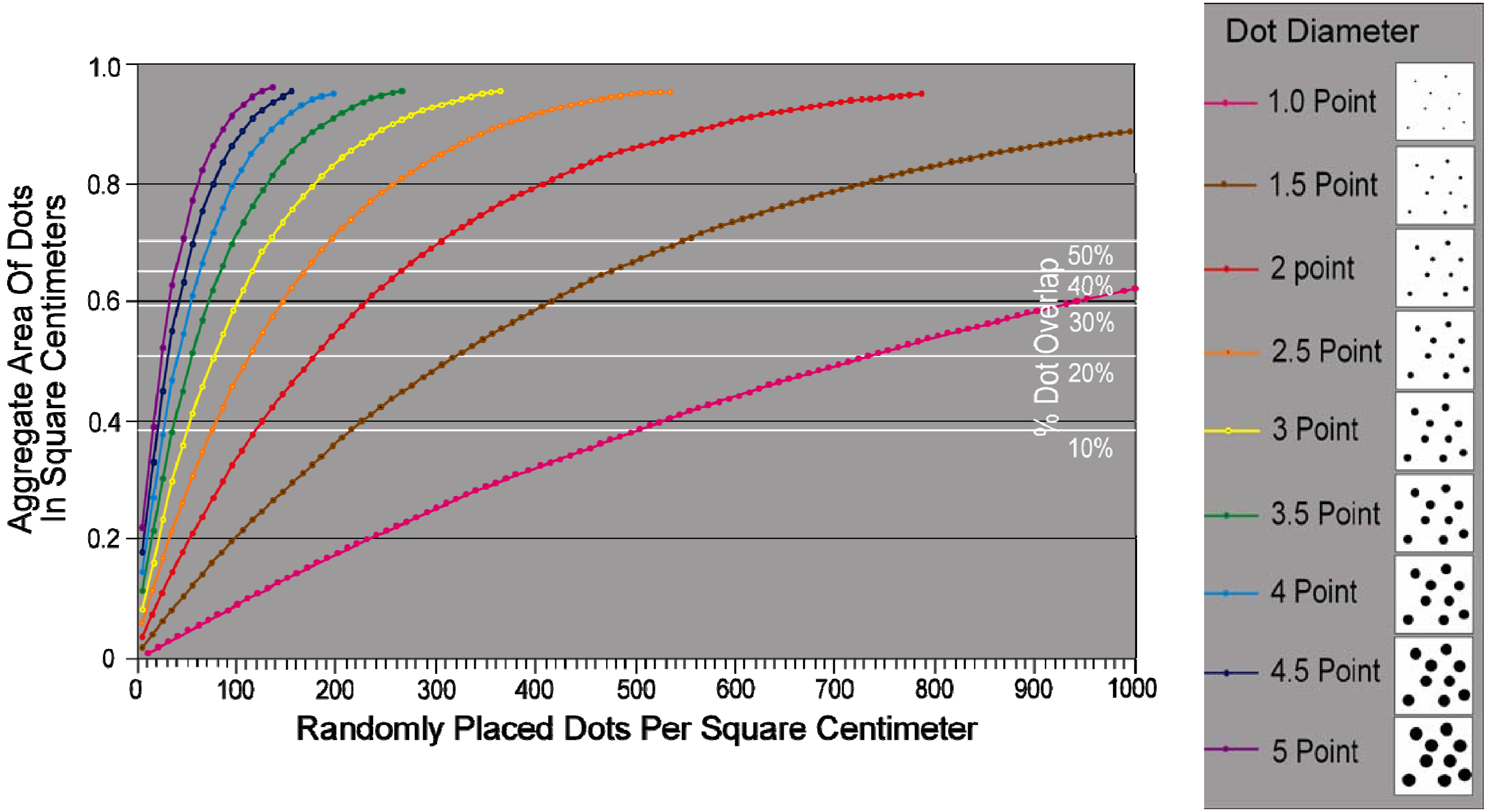
Running the equation with ESRI dots in 10 dot increments gave the results plotted on this graph.



Equal overlap points define lines of constant dot overlap – a more precise form of zone of coalescing dots.



Testing the model…



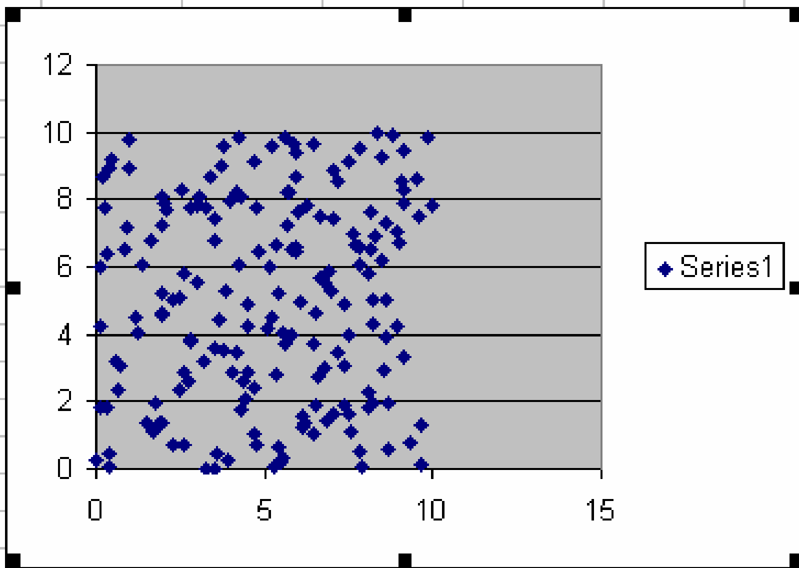
Step 1. Generate lots of random numbers from 0.0 mm to 10.0 mm.

```
#include <stdio.h>
#include <stdlib.h>

main()
{

        char oname[80];
   FILE*ofil;
        int seed;

        double r; /* random value in range [0,1) */

        int count;
        seed = 10000; /* choose a seed value */
        srand(seed); /*initialize random number generator*/

        printf("Enter output file name: ");
  scanf ("%s",oname);
  if((ofil = fopen(oname,"wt")) == NULL)
  {
    printf("error: cannot open output file\n");
    return 1;
  }

  for(count=1; count<=3000; ++count)
  {
   r = 10.0*(rand() / ((RAND_MAX)+(1.0)));
   fprintf(ofil,"%lf\n",r);
  }
  return 0;
} /*main*/
```

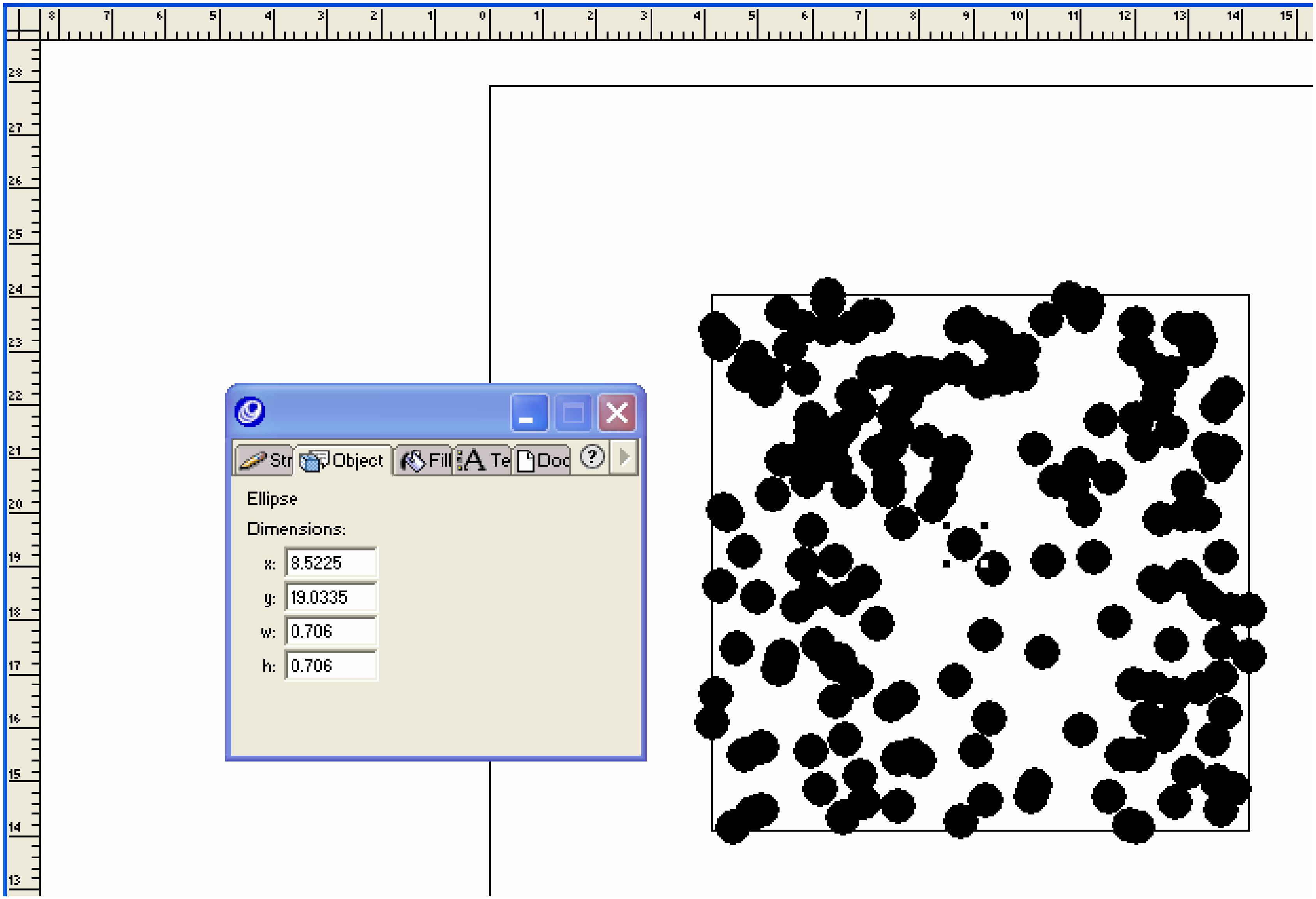
9.977417
3.888855
3.605957
6.149292
0.285339
7.028809
0.999451
6.518555
2.563782
6.928406
0.412292
2.995300
2.835388
5.465698
1.372375
6.512756
4.749756
7.342224
7.826538
6.076050
3.758240

Step 2. Split the numbers into two Excel columns and make a scatterplot.

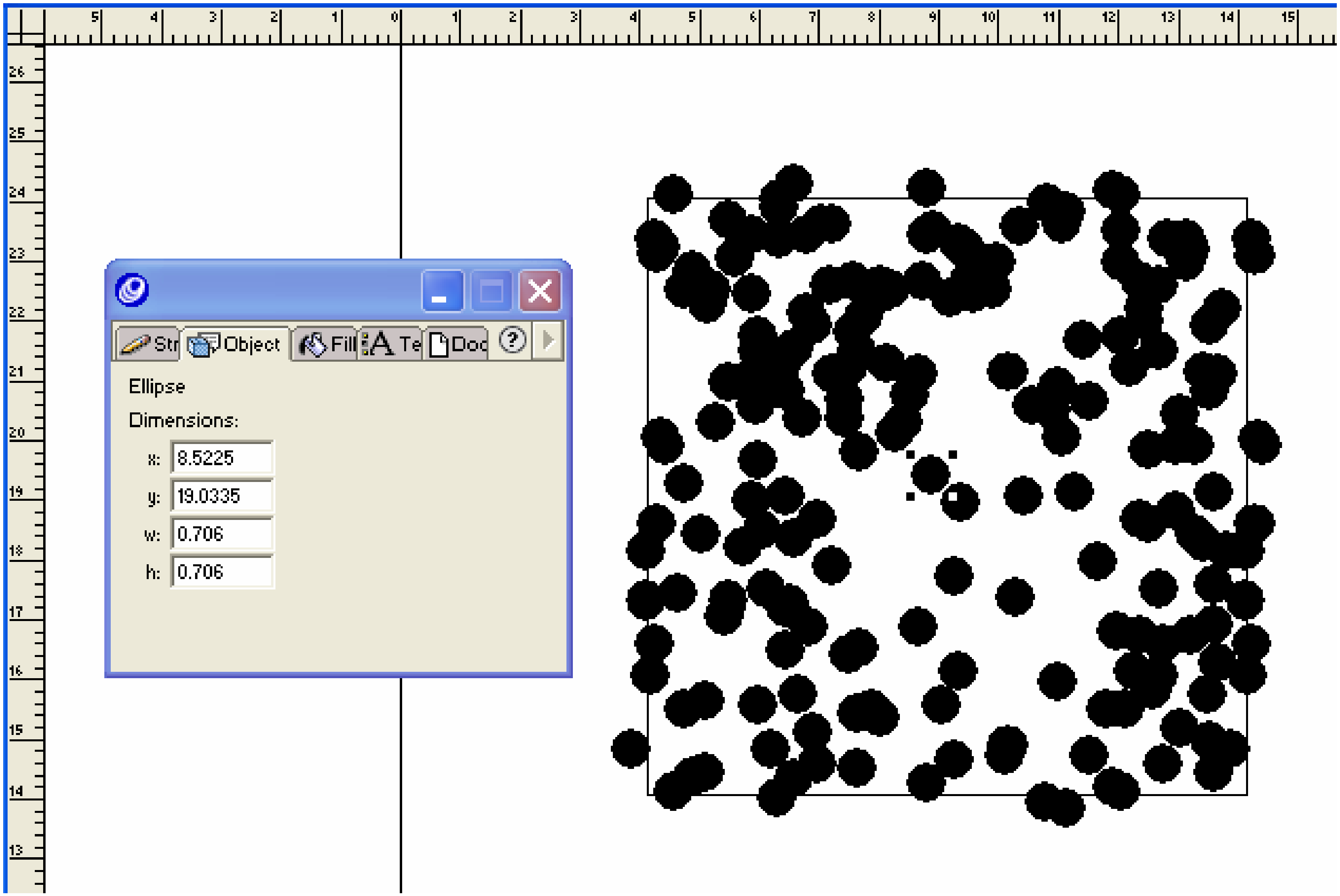
	A	B
1	9.977417	7.82135
2	3.888855	0.230713
3	3.605957	0.48645
4	6.149292	1.208496
5	0.285339	7.751465
6	7.028809	7.460632
7	0.999451	8.928528
8	6.518555	1.913452
9	2.563782	8.267212
10	6.928406	5.852966
11	0.412292	0.43457
12	2.9953	7.832642
13	2.835388	7.731934
14	5.465698	0.187683
15	1.372375	6.061707
16	6.512756	4.609985
17	4.749756	7.773132
18	7.342224	4.902649
19	7.826538	9.530945
20	6.07605	4.972839
21	3.75824	3.533325
22	3.805542	9.594116
23	8.224182	1.980591
24	4.75769	0.725708
25	0.372925	8.94043
26	9.124756	9.457092
27	7.341309	1.894531
28	0.482178	9.216309
29	9.069519	8.55835
30	0.132751	5.978699
31	8.902588	4.267273



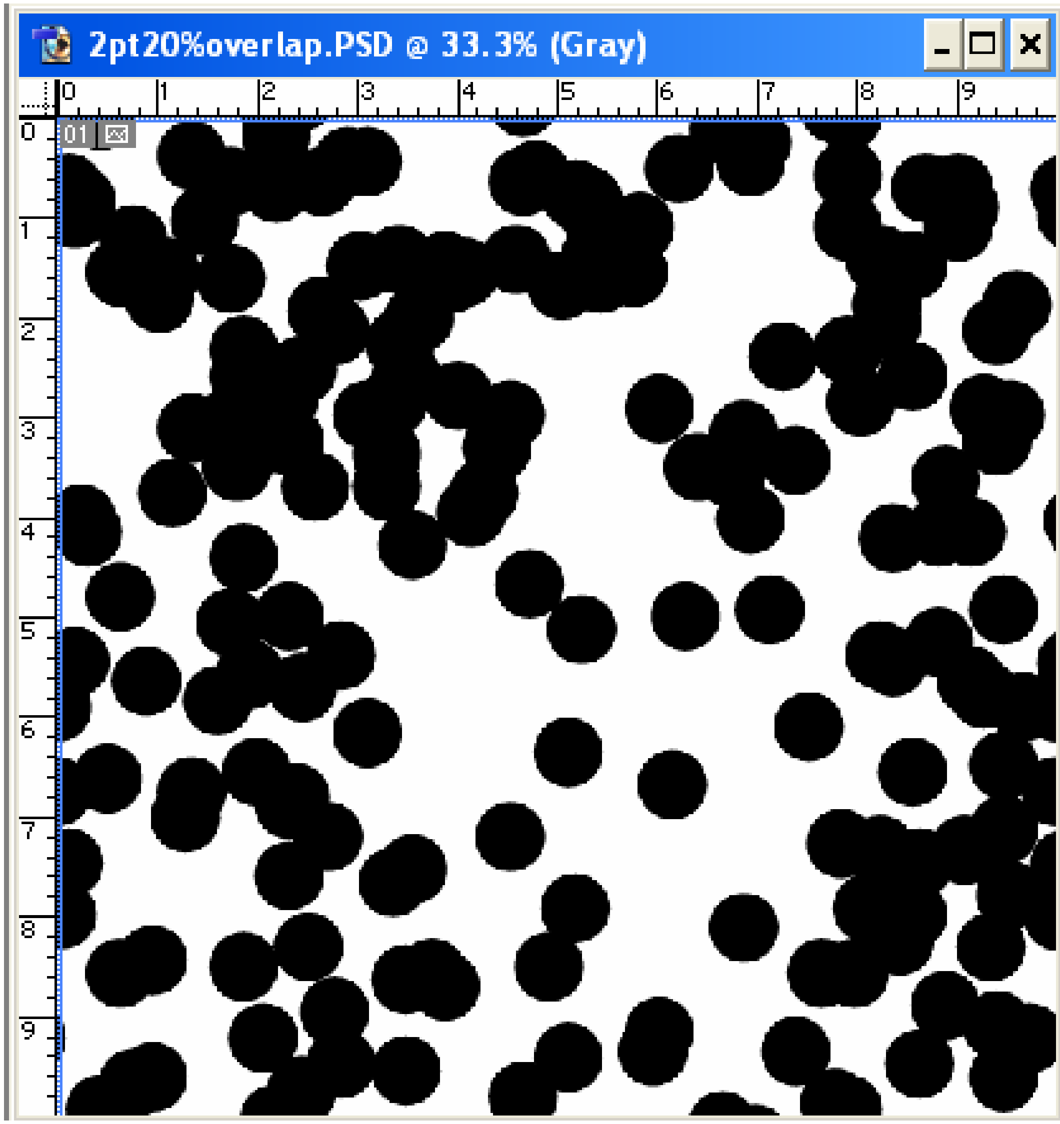
Step 3. Import the scatterplot into Freehand and change the dots to the desired diameter.



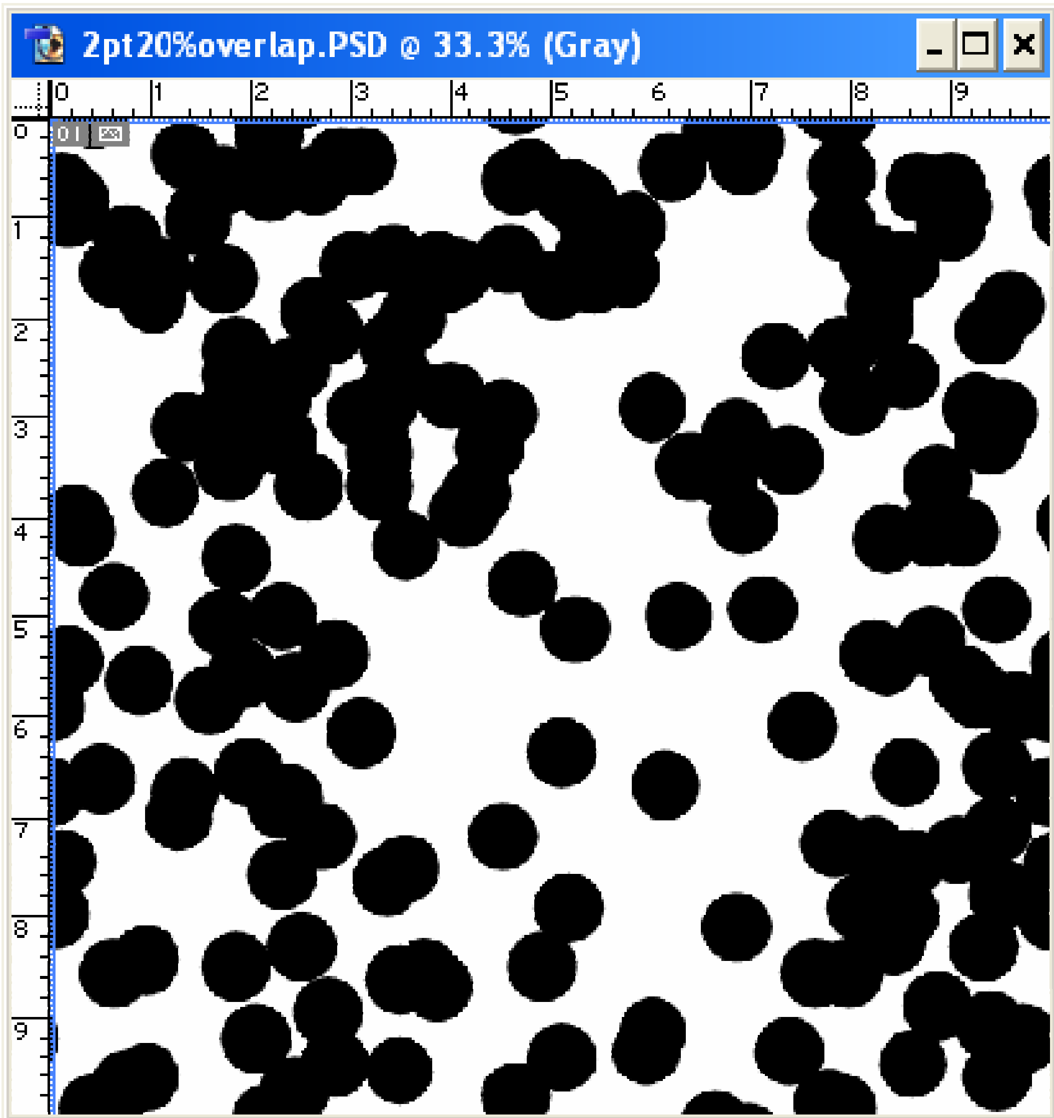
Step 4. Wrap the dots around the four edges.

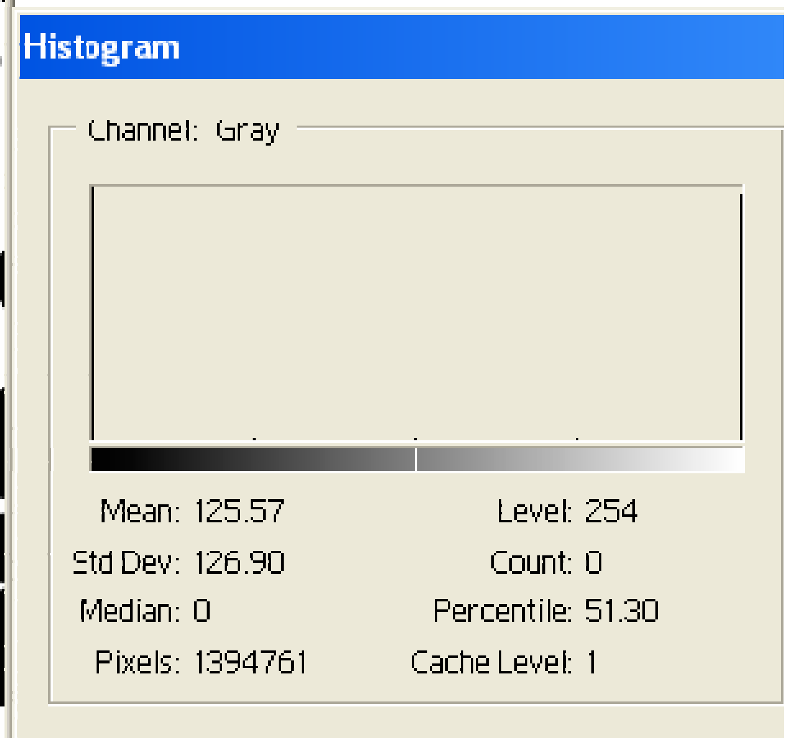


Step 5. Import the graphic into Photoshop and crop the edge dots.

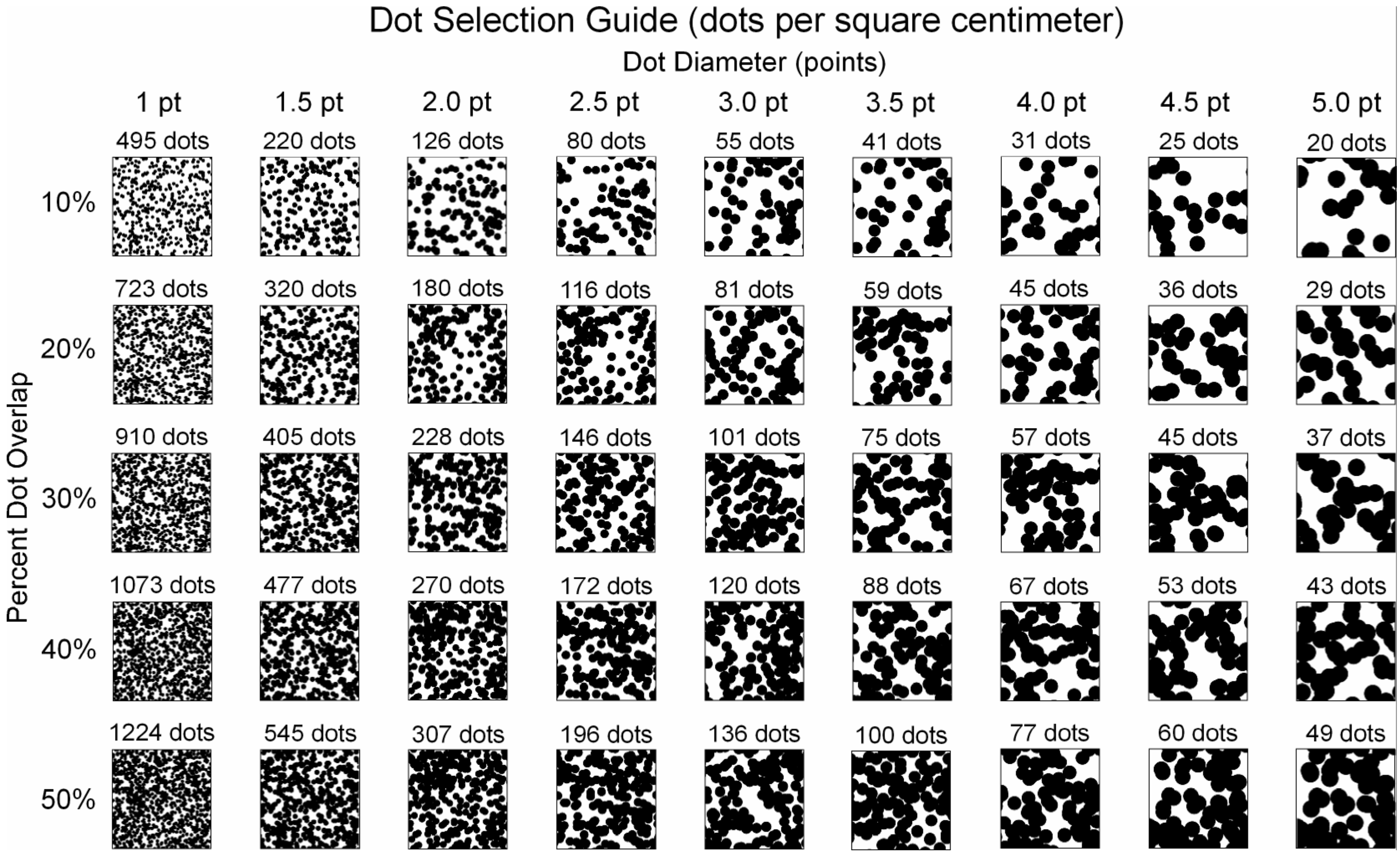


Step 6. Use the histogram tool to find the proportion of the square covered by dots.

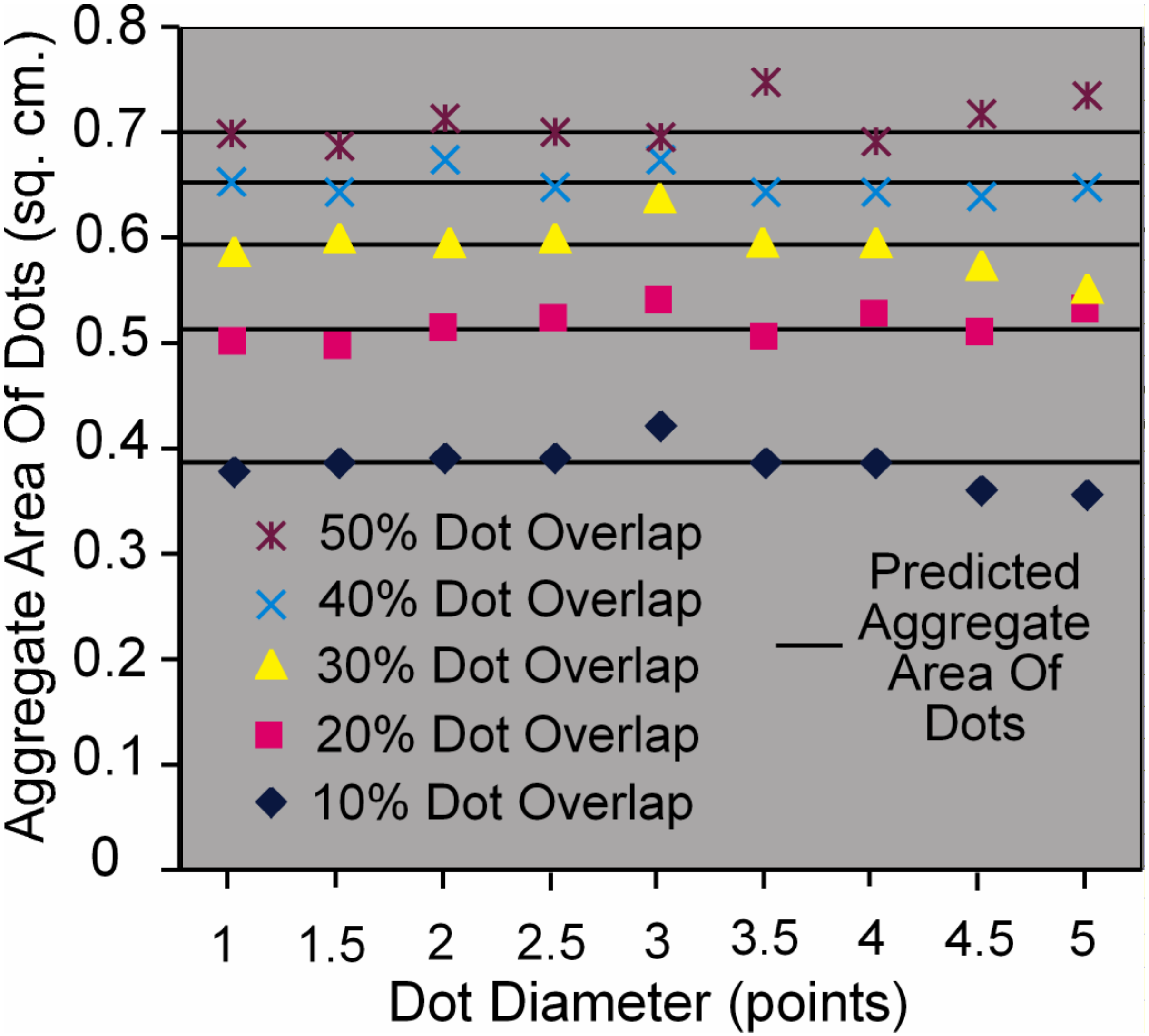




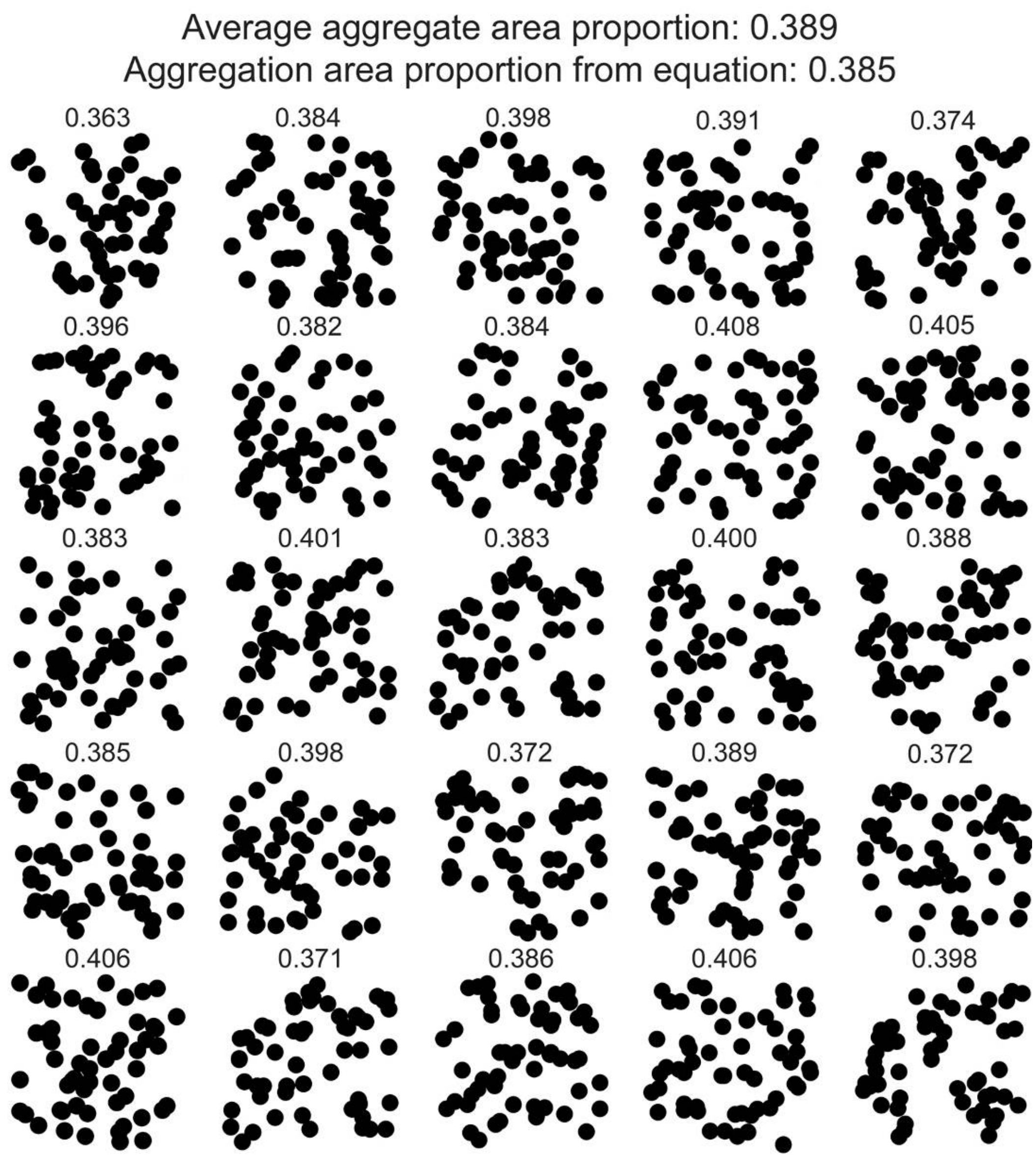
I made these dot proportion measurements for the number of dots predicted at different point sizes for 10 - 50% dot overlap.



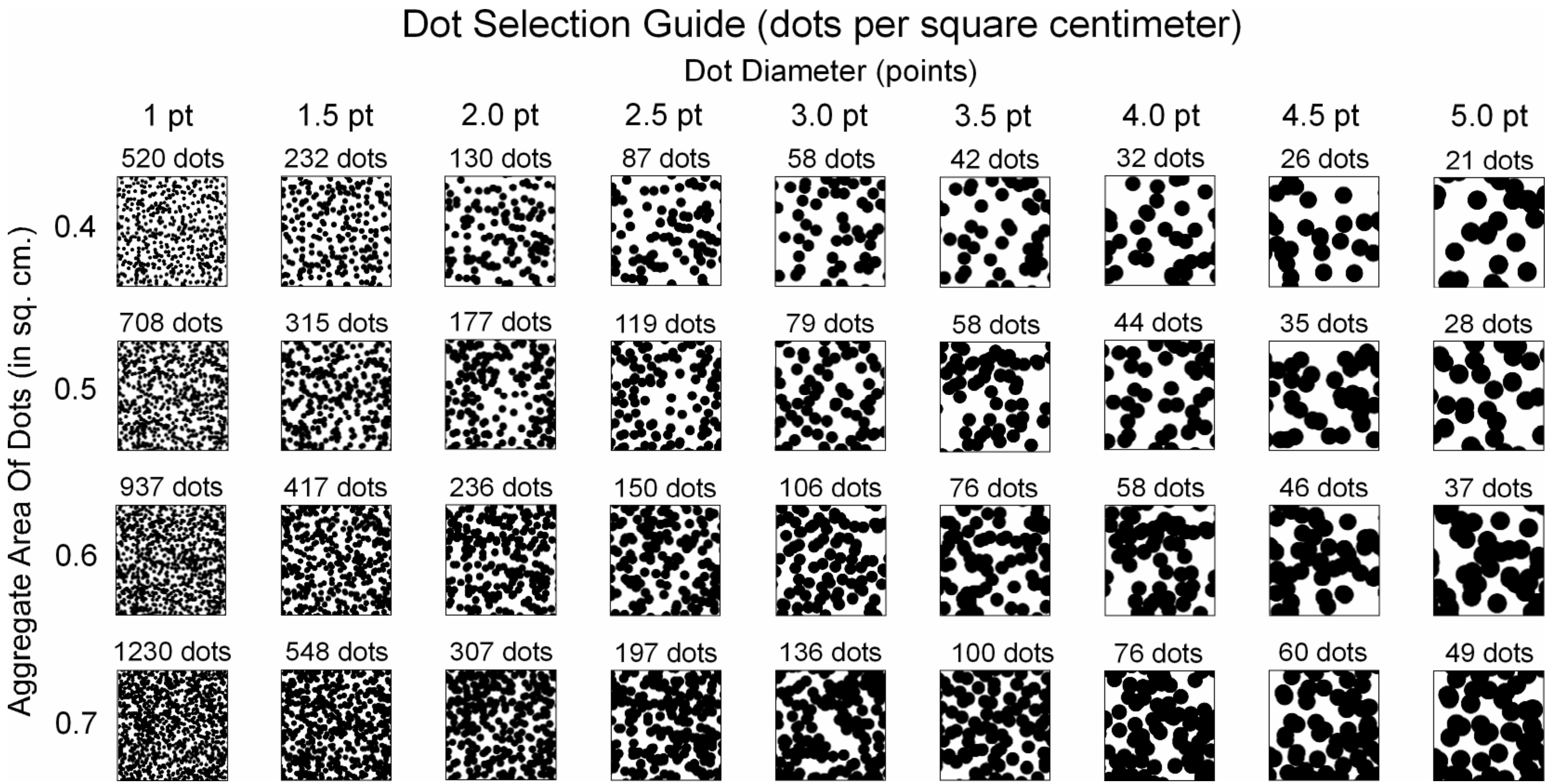
How close did the measured aggregate areas match those predicted from the Unification Equation?



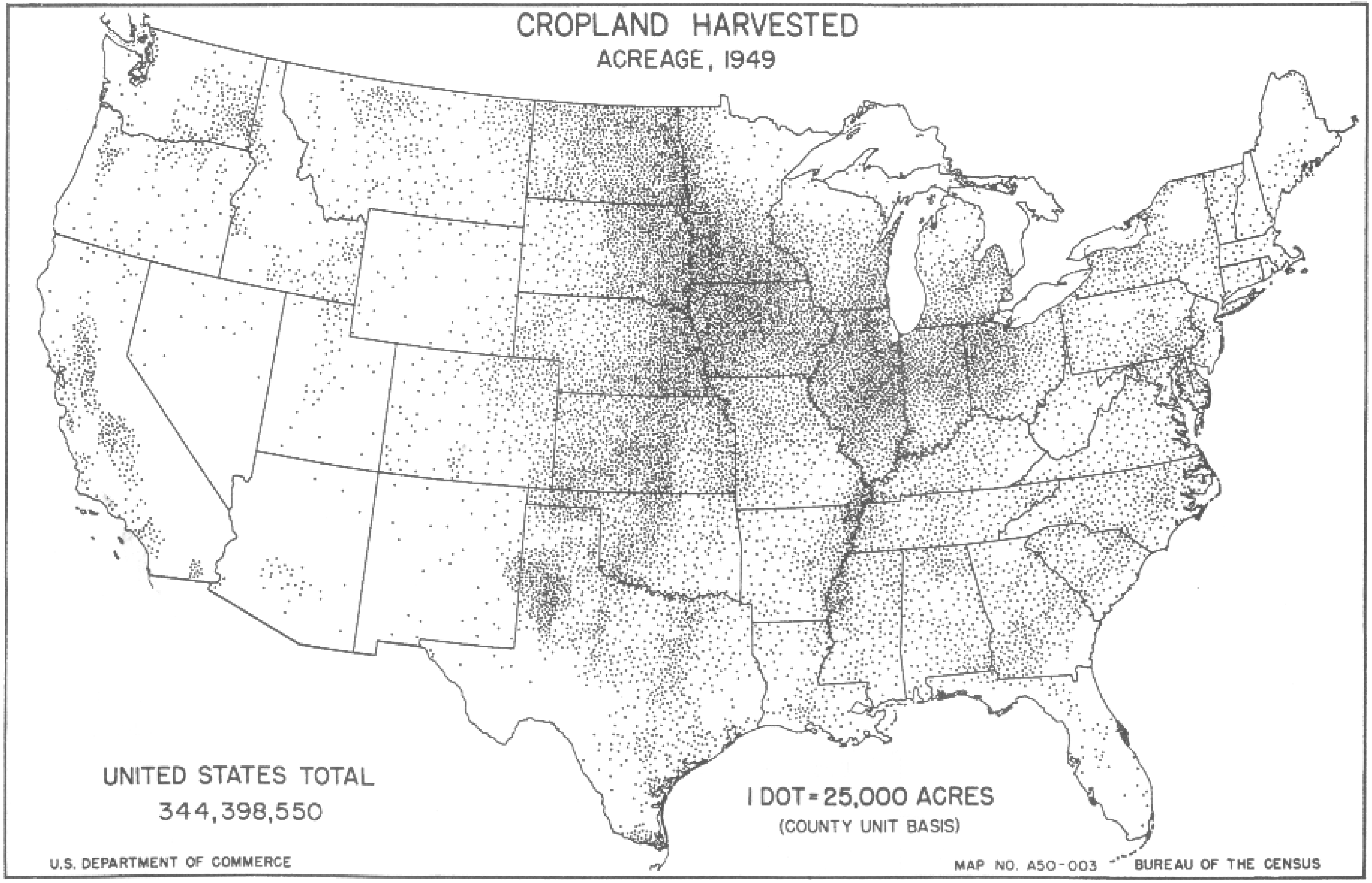
I tested the equation for 10% overlap of 3 pt dots – 55 random dots.



I also made a Dot Selection Guide based on aggregate dot area.

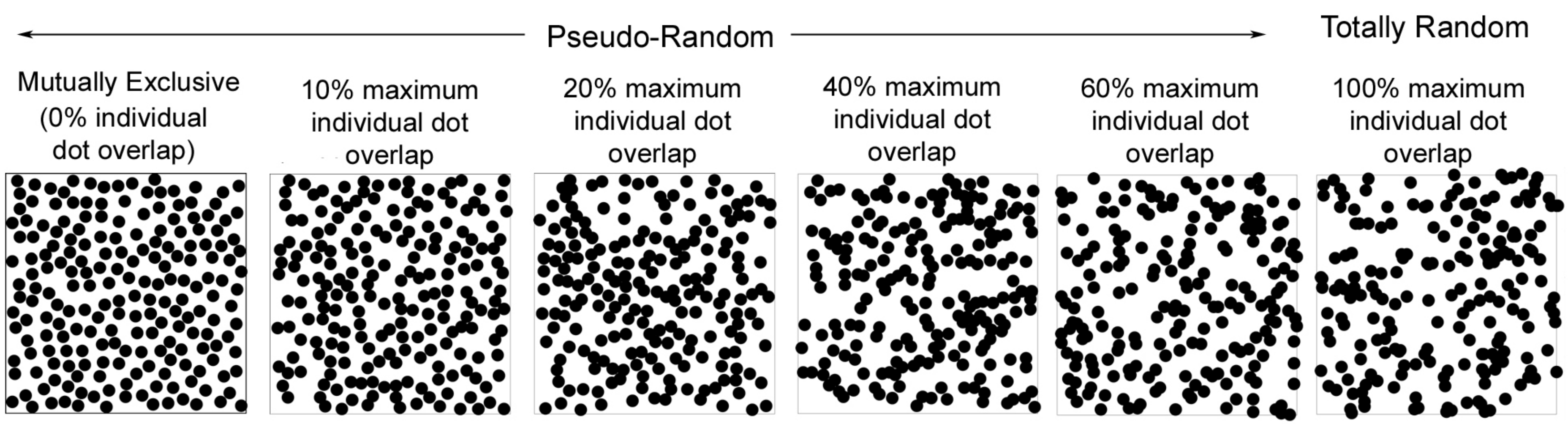


What is the problem with random dot placement?



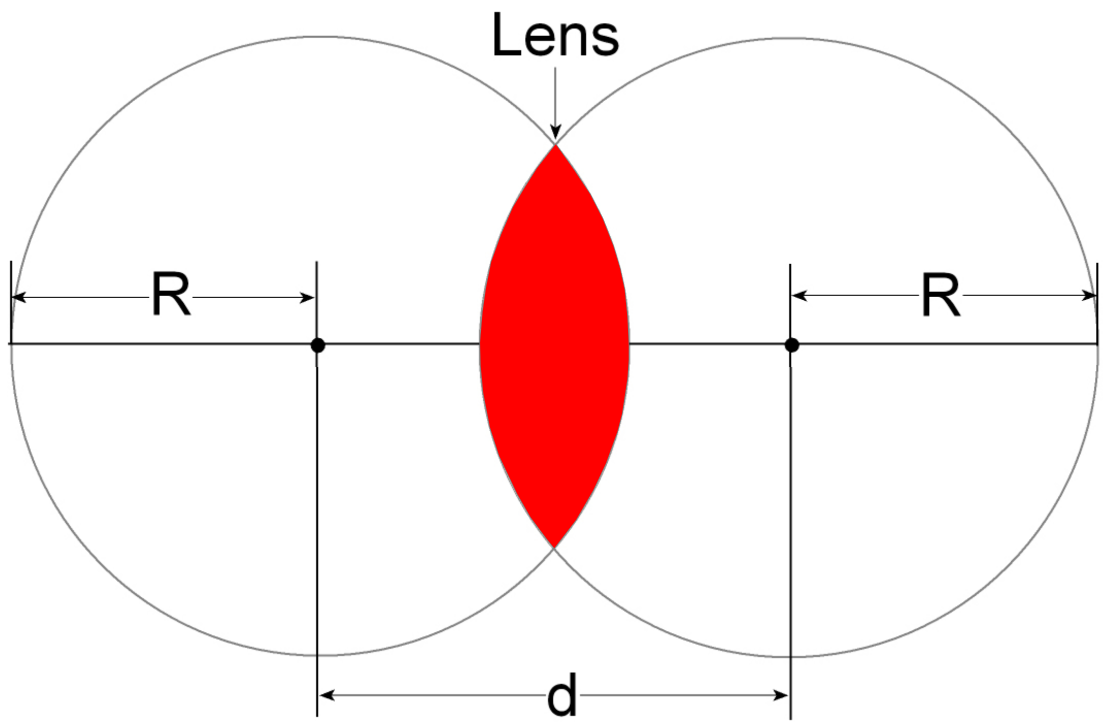
Cartographers place dots manually in a pseudo-random fashion!

Let's look at pseudo-randomness in terms of maximum allowable overlap of individual dots…



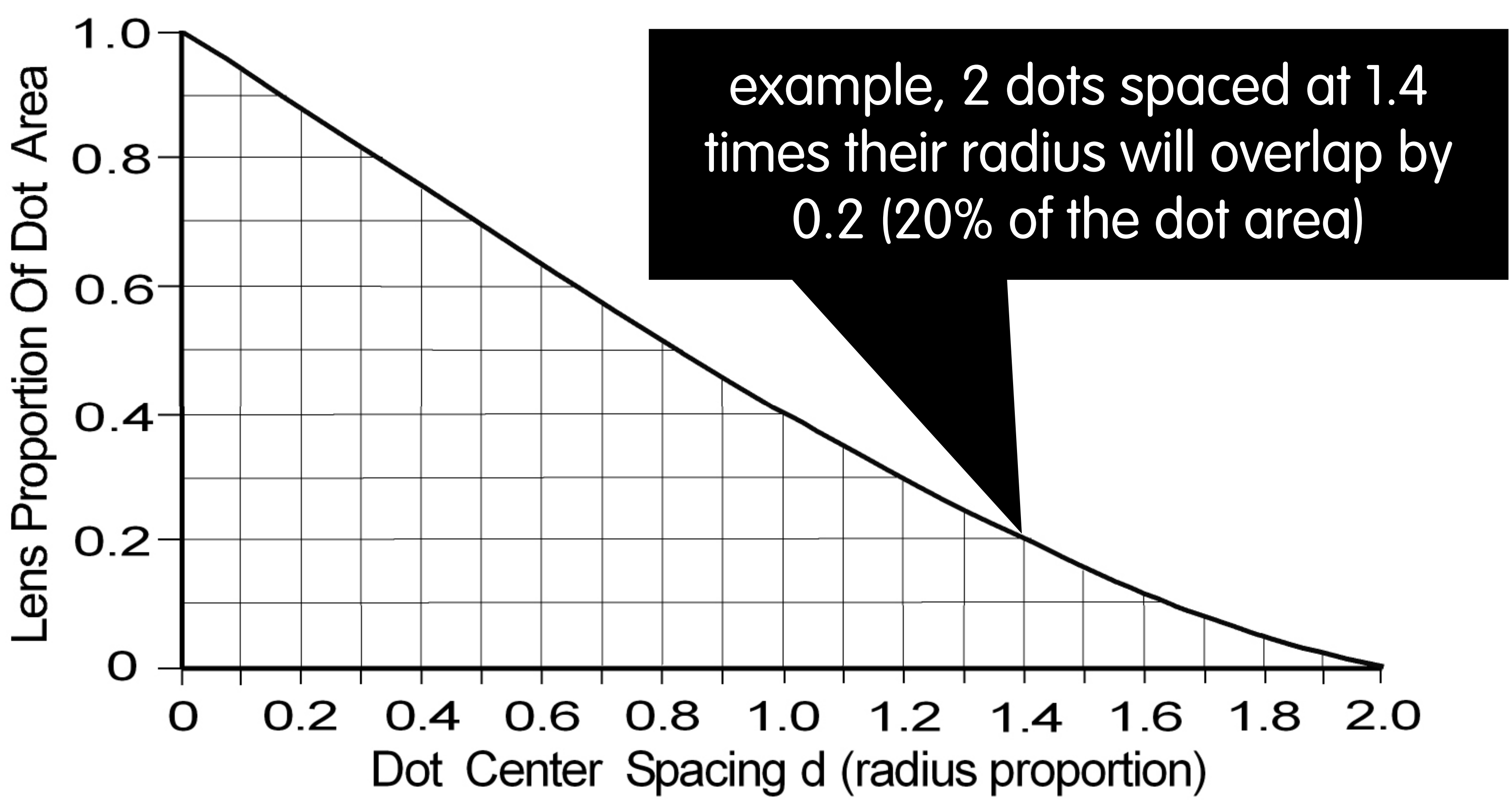
200 dots are placed in each square

Individual dot overlap is computed from the lens area equation.



$$Area_{Lens} = 2R^2 \cos^{-1}\left(\frac{d}{2R}\right) - \frac{1}{2} d\sqrt{4R^2 - d^2}$$

Graphing the lens area equation shows the non-linear relationship between the spacing of dot centers and the proportion of a dot overlapped by the lens.

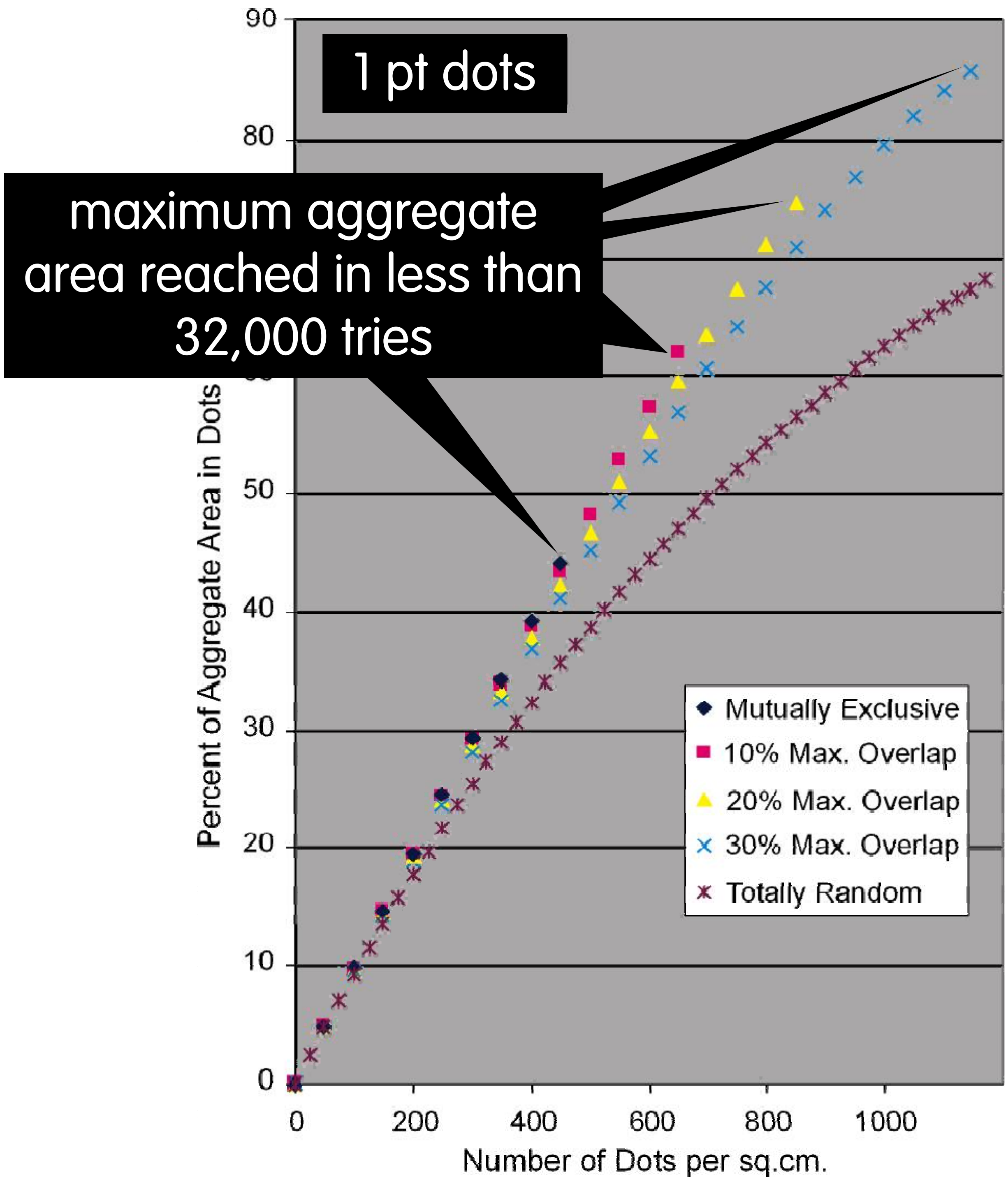


This information is the basis for a pseudo-random dot generator.

1. Select a dot radius and maximum dot overlap.
2. Generate the first random dot position.
3. For succeeding dots, compute the distance between the dot center and all other dot centers.
4. If all distances are greater than d, add the dot to the array of dots – otherwise, discard the dot position and generate another random position.
5. Repeat until the number of dots you need is generated, or until a maximum number of tries is reached.

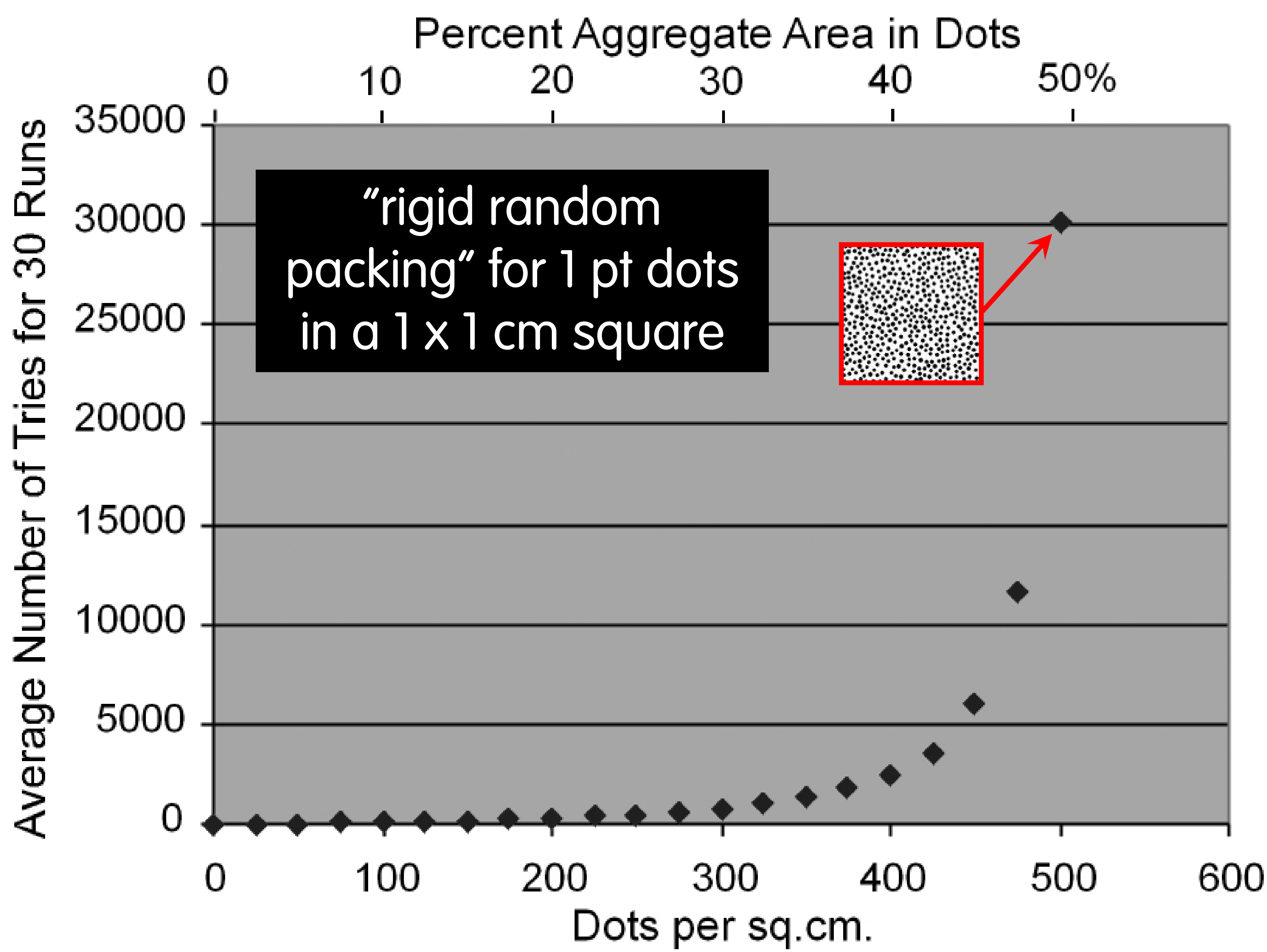
The procedure…

- I wrote a C program to run the procedure 30 times for 10%, 20%, and 30% maximum individual dot overlaps, in steps of 25 or 50 dots with a maximum of 32,000 tries for each run.
- Once I had a pseudo-random dot position array, I could check the distance of each dot against all other dots, and compute the lens area if a dot was less than a dot diameter away.
- Summing the lens areas gave the total dot overlap area, and hence the aggregate proportion or percentage covered by dots, assuming that there were no triple dot overlaps.

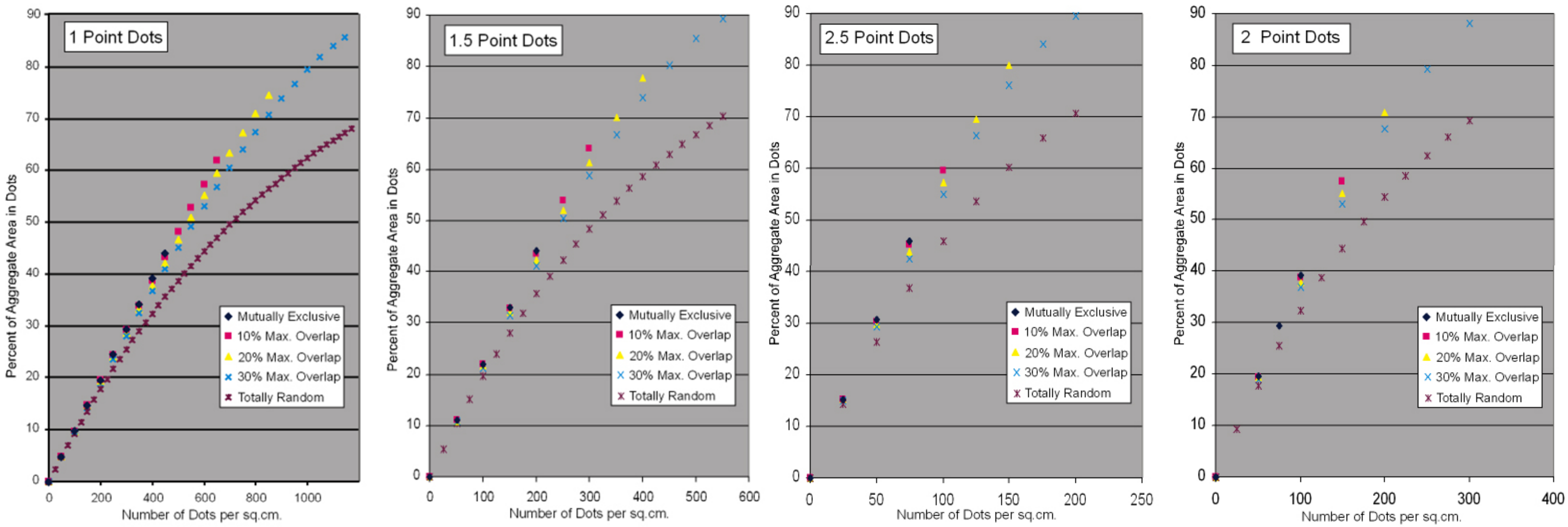


1 pt dots
maximum aggregate area reached in less than 32,000 tries
Percent of Aggregate Area in Dots
Number of Dots per sq.cm.
Mutually Exclusive
10% Max. Overlap
20% Max. Overlap
30% Max. Overlap
Totally Random

What is happening with mutually exclusive dot placement?



Do the graphs for 1.0 pt to 2.5 pt dots look similar?



Aggregate area equations…

Mutually exclusive dots:

$$Area = np$$

Totally random dots:

$$Area = np - \frac{n!}{2!(n-2)!}p^2 + \frac{n!}{3!(n-3)!}p^3 - \ldots + (-1)^{10}\frac{n!}{10!(n-10)!}p^{10}$$

My guess is that the equation for intermediate pseudo-random dots is a linear combination of the two bounding equations above.

A general aggregate area equation…

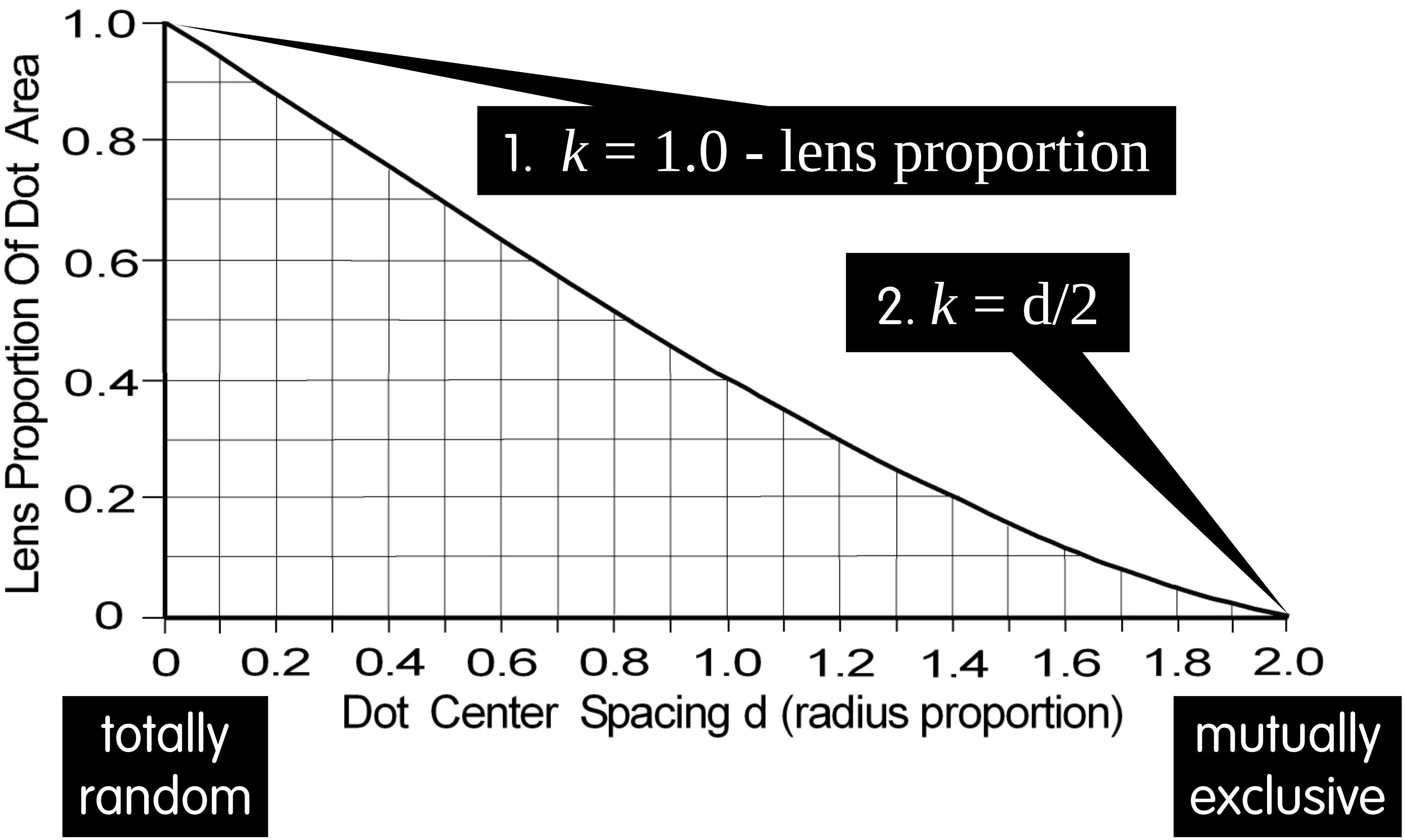
$$Area = knp + (1-k)\left(np - \frac{n!}{2!\,(n-2)!} p^2 + \frac{n!}{3!\,(n-3)!} p^3 - \ldots + (-1)^{10} \frac{n!}{10!\,(n-10)!} p^{10} \right)$$

or

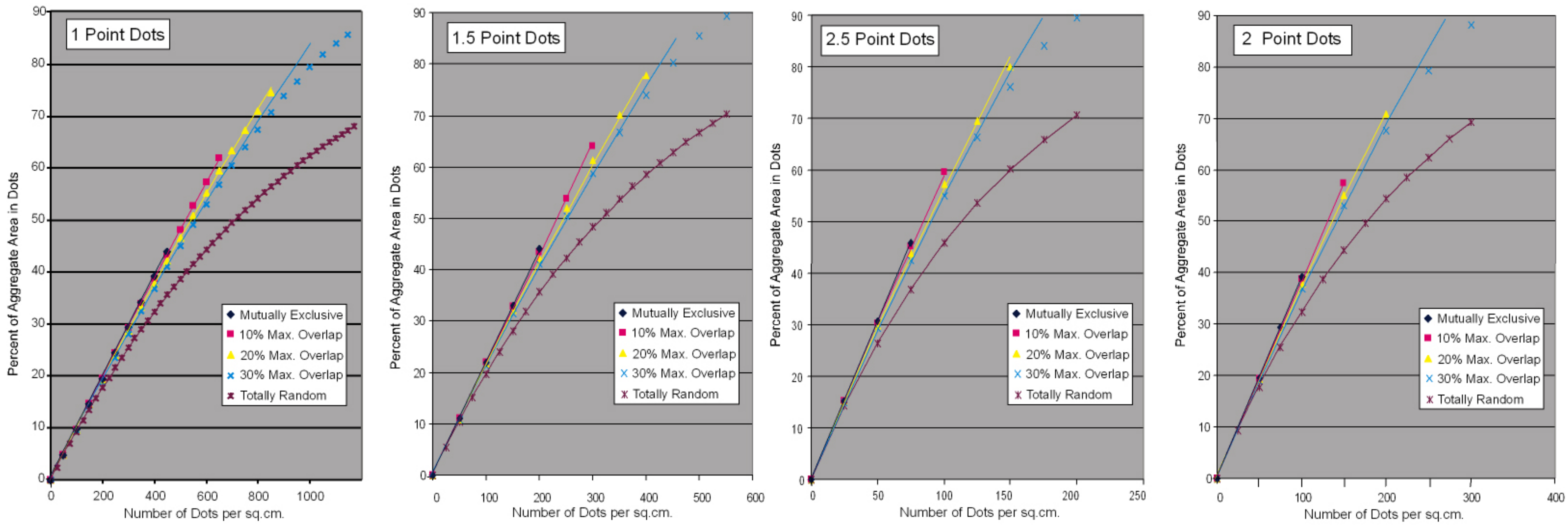
$$Area = np + (1-k)\left(- \frac{n!}{2!\,(n-2)!} p^2 + \frac{n!}{3!\,(n-3)!} p^3 - \ldots + (-1)^{10} \frac{n!}{10!\,(n-10)!} p^{10} \right)$$

where k ranges from 0 (totally random) to 1 (mutually exclusive).

Guesses as to what k is proportional to?

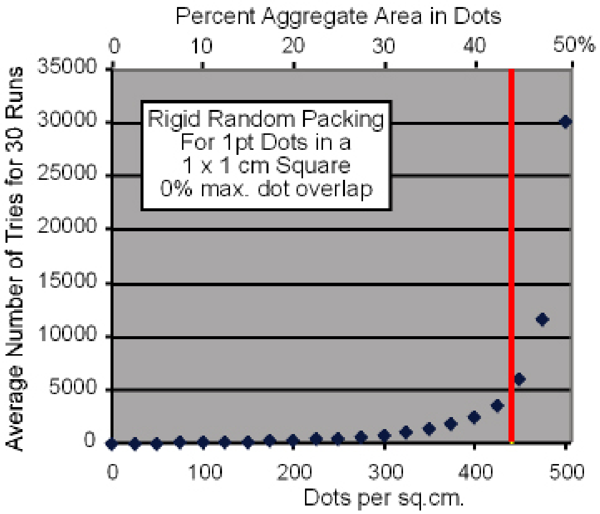


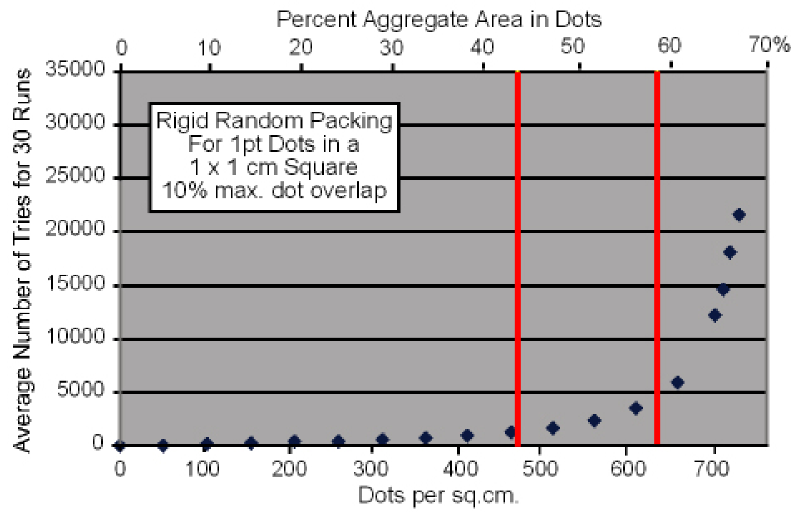
The second possibility for k fit the data better – about twice as good a fit.

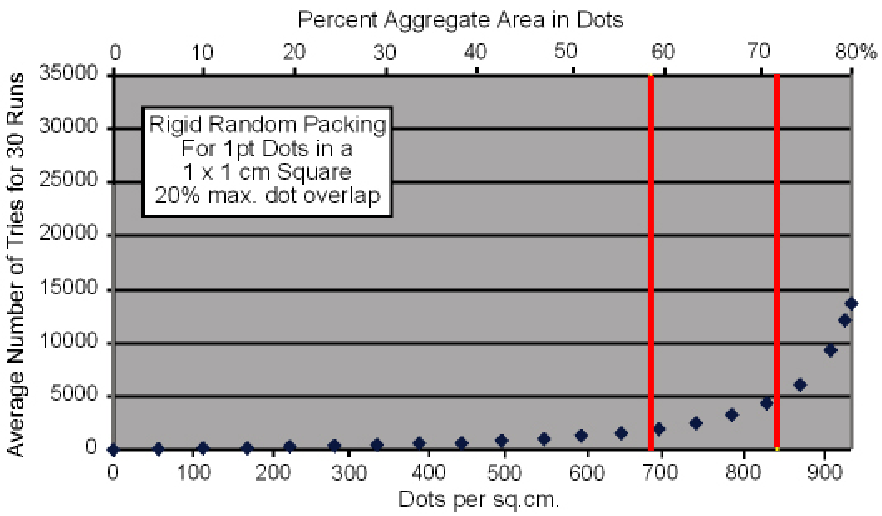


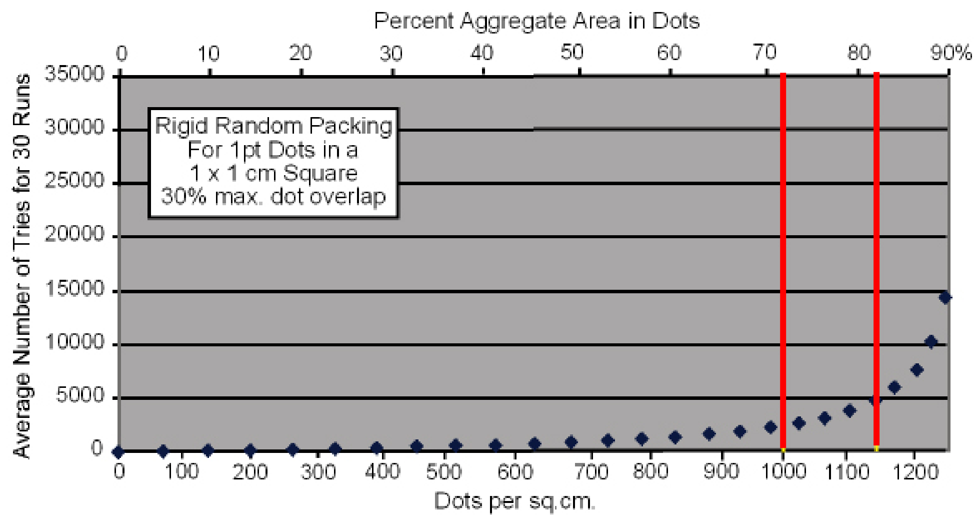
Average values for a number of tries of 30 runs of creating pseudo-random are plotted on these graphs.

1 pt dot example

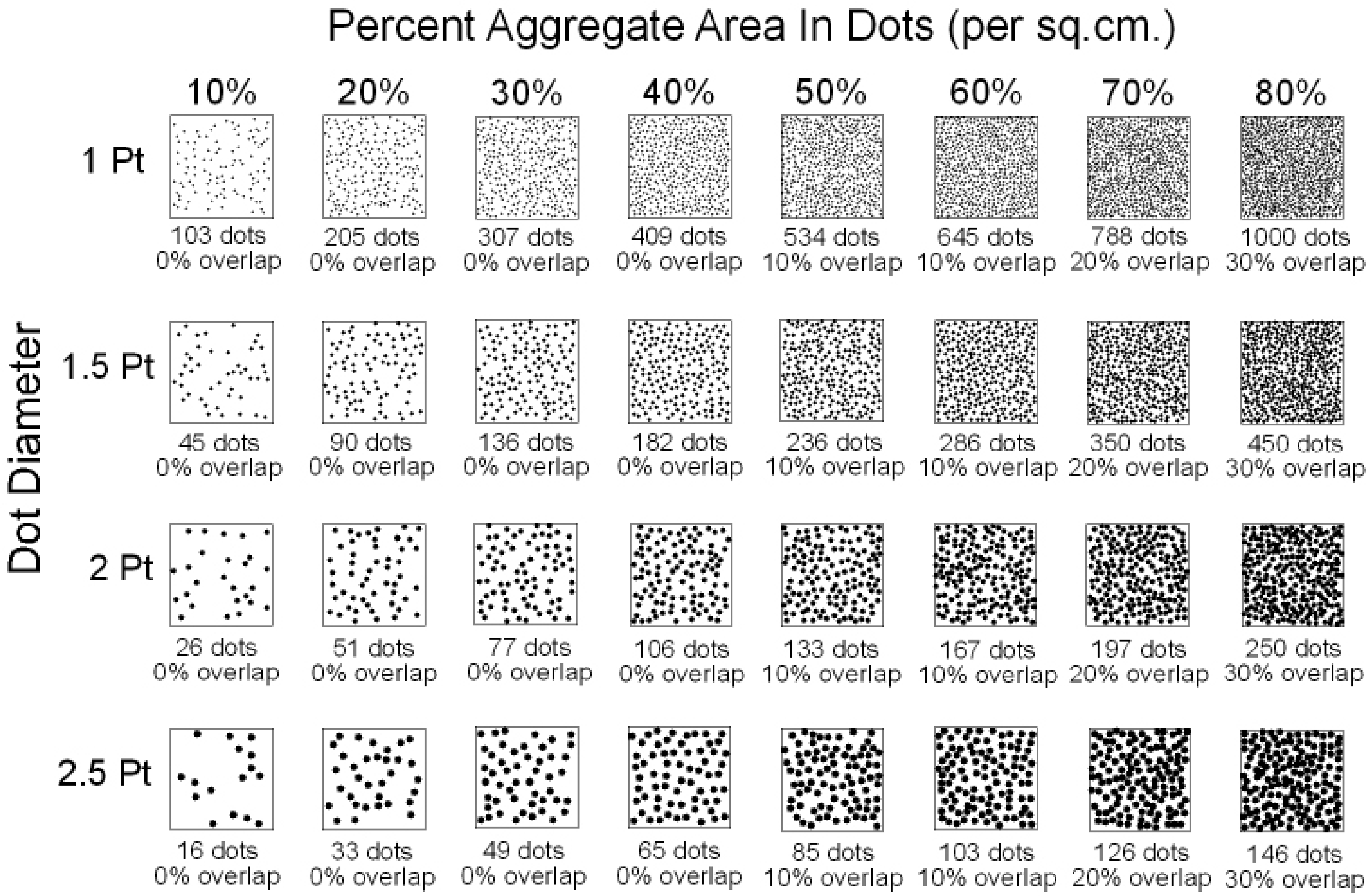




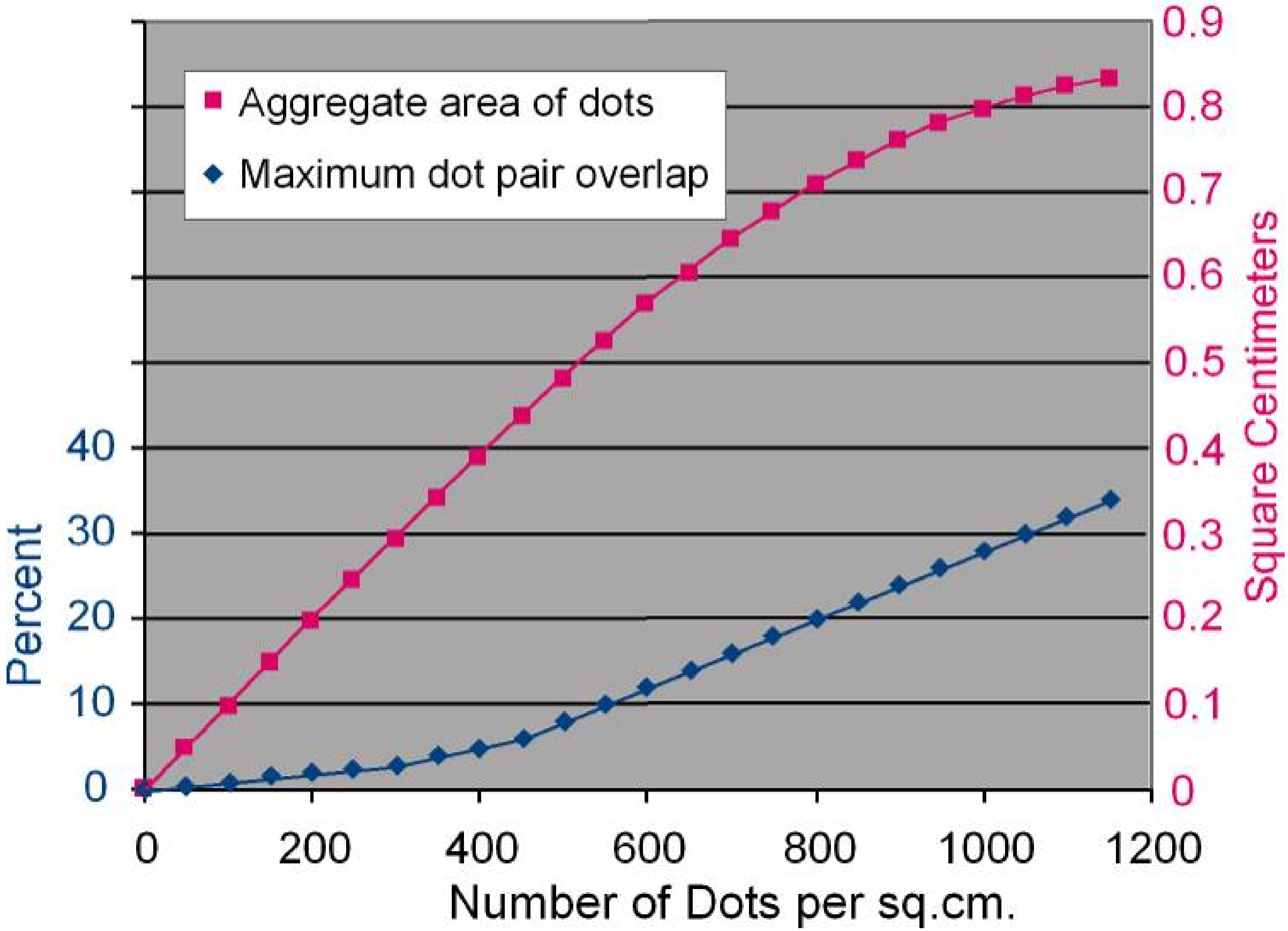




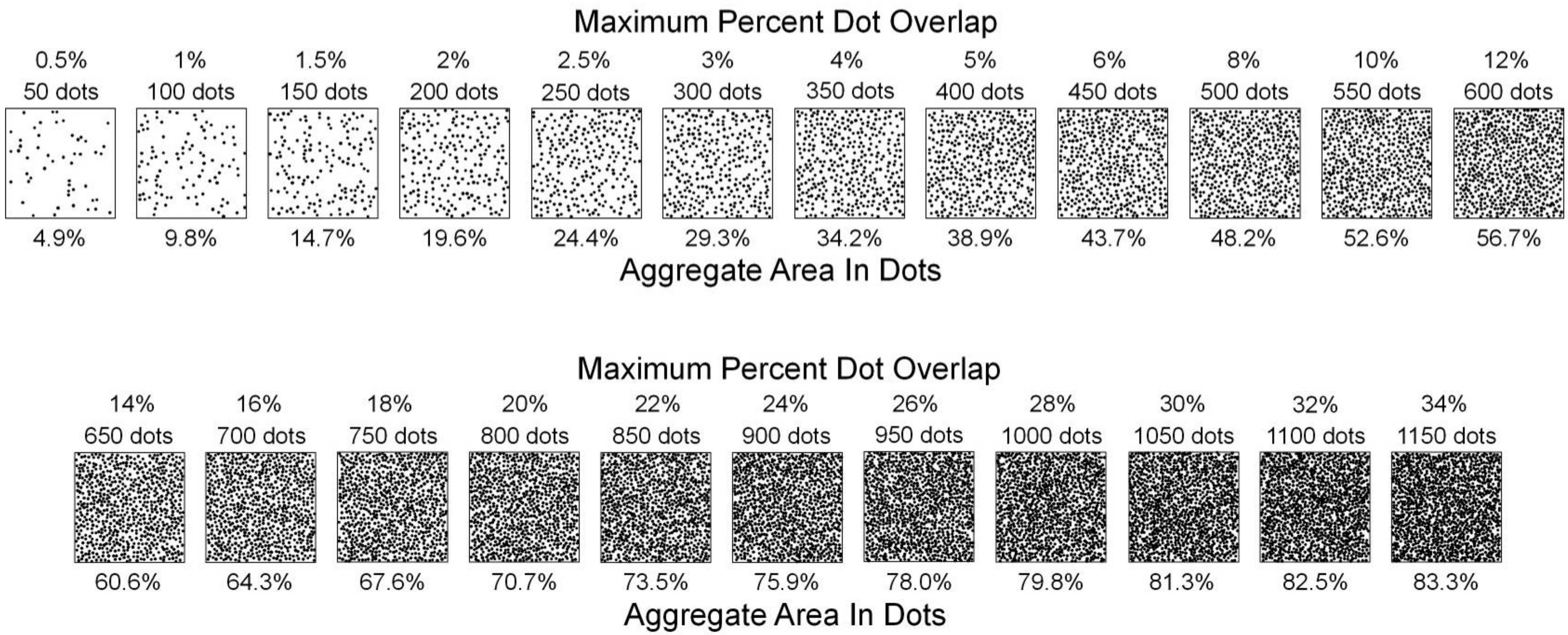
From these values we can make a pseudo-random dot selection guide.

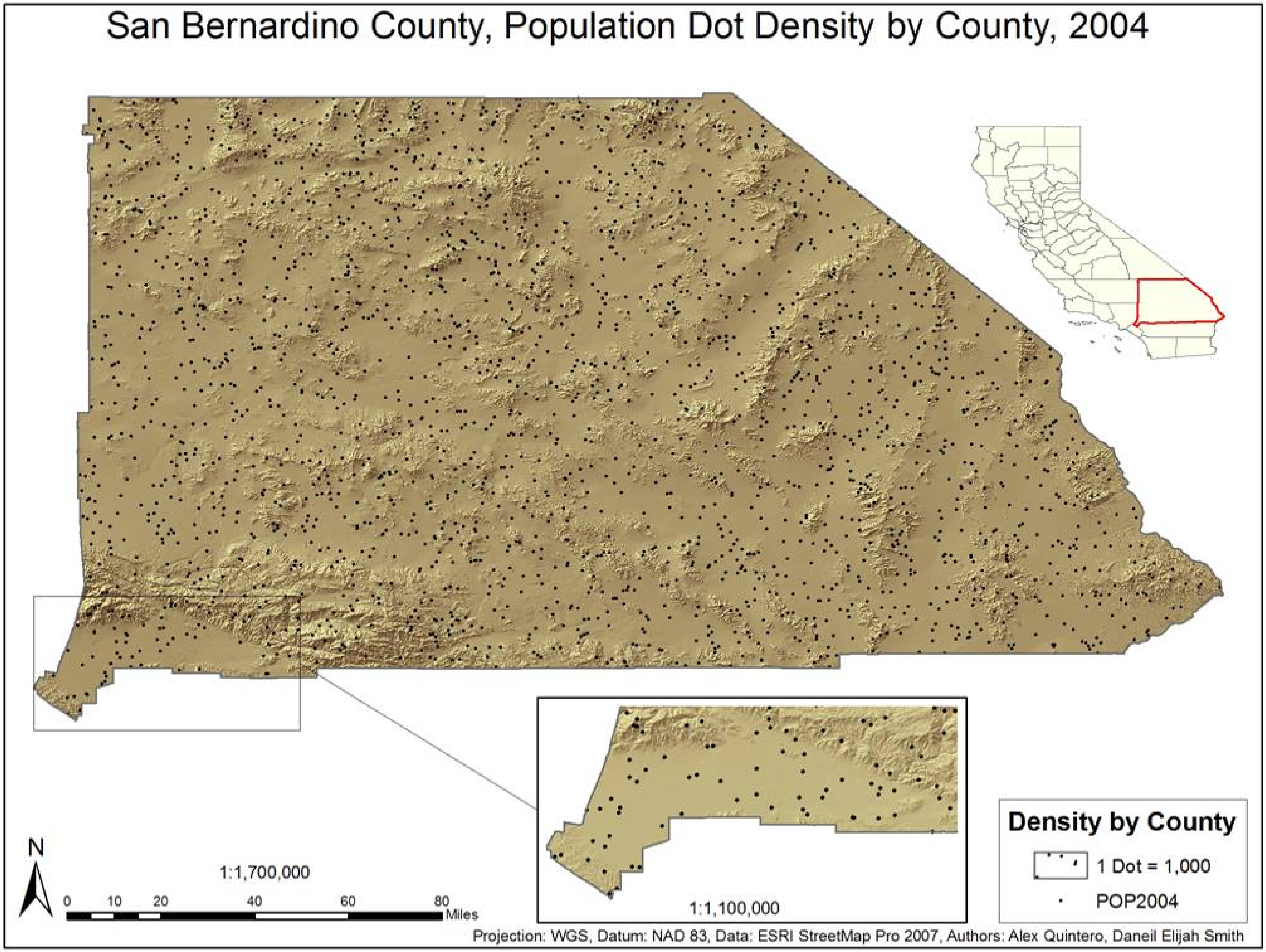


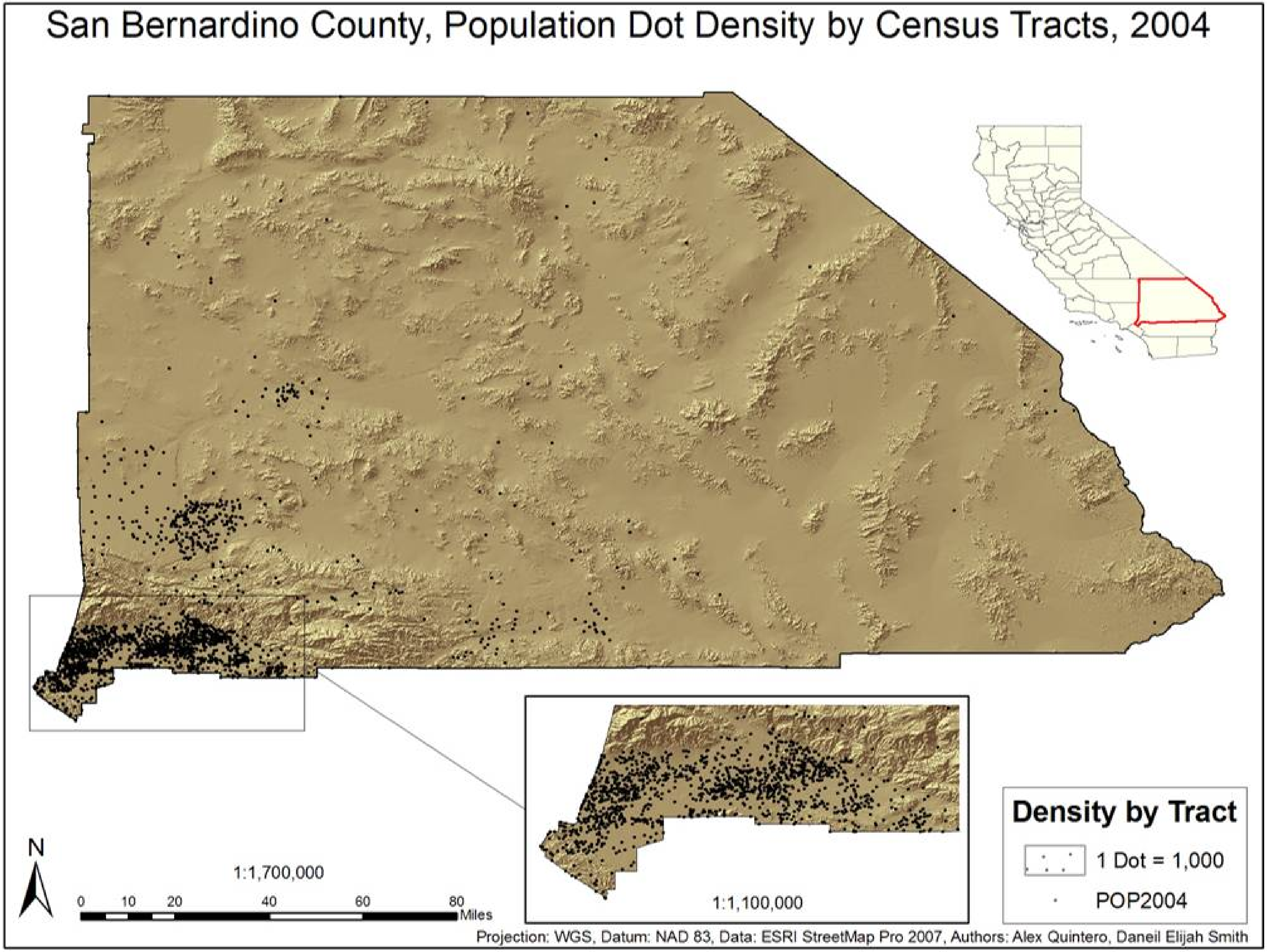
Pseudo-random placement of 1 pt dots with less than 5,000 tries…



Pseudo-random Dot Selection Guide for 1 pt dots with variable maximum dot overlap…

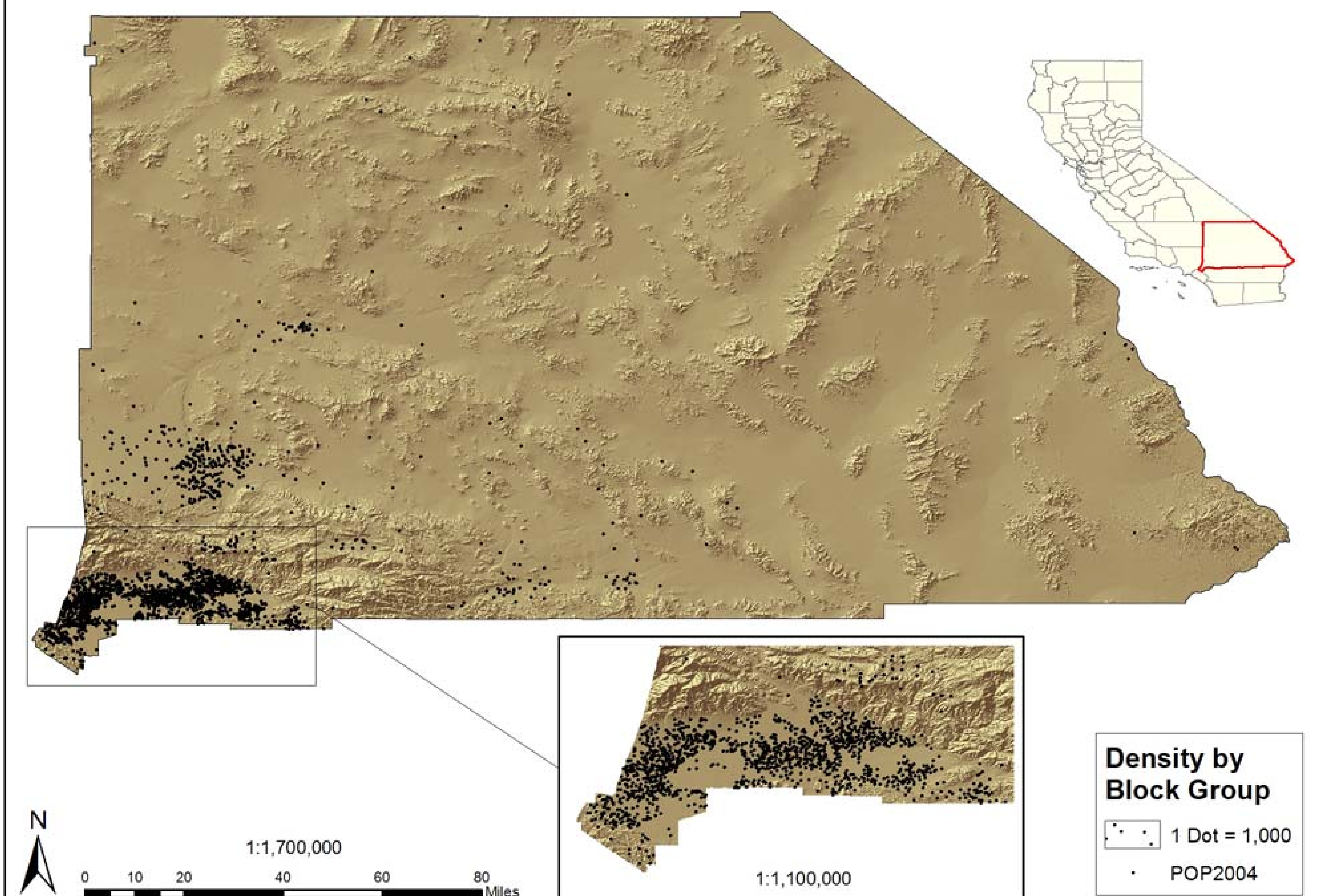



San Bernardino County, Population Dot Density by County, 2004
Density by County
1 Dot = 1,000
POP2004
N
1:1,700,000
0
10
20
40
60
80
Miles
1:1,100,000
Projection: WGS, Datum: NAD 83, Data: ESRI StreetMap Pro 2007, Authors: Alex Quintero, Daneil Elijah Smith

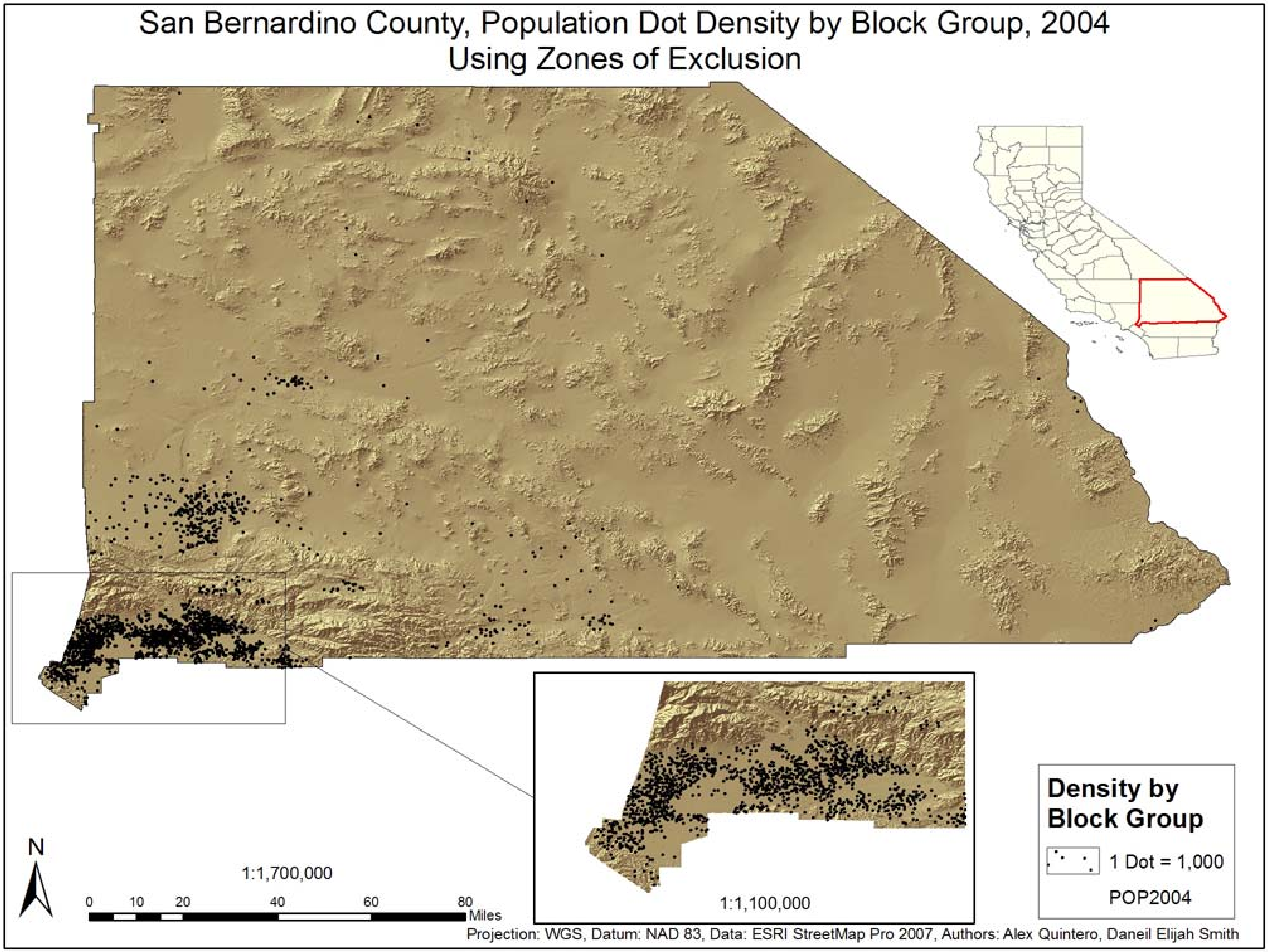


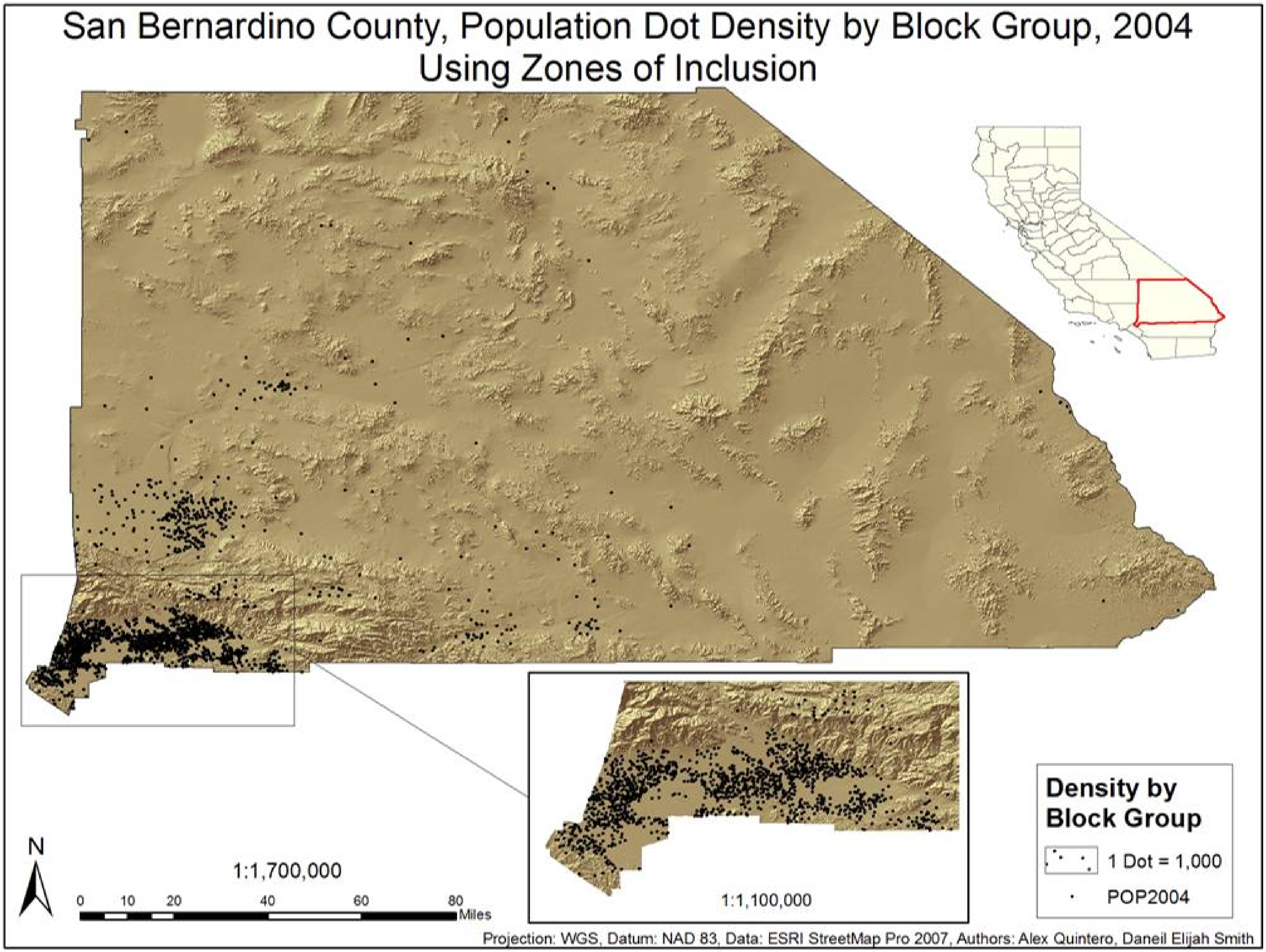
San Bernardino County, Population Dot Density by Census Tracts, 2004
Density by Tract
1 Dot = 1,000
POP2004
N
1:1,700,000
0 10 20 40 60 80
Miles
1:1,100,000
Projection: WGS, Datum: NAD 83, Data: ESRI StreetMap Pro 2007, Authors: Alex Quintero, Daneil Elijah Smith

San Bernardino County, Population Dot Density by Block Group, 2004



Projection: WGS, Datum: NAD 83, Data: ESRI StreetMap Pro 2007, Authors: Alex Quintero, Daneil Elijah Smith


San Bernardino County, Population Dot Density by Block Group, 2004
Using Zones of Exclusion
1:1,100,000
Density by Block Group
1 Dot = 1,000
POP2004
N
1:1,700,000
0 10 20 40 60 80
Miles
Projection: WGS, Datum: NAD 83, Data: ESRI StreetMap Pro 2007, Authors: Alex Quintero, Daneil Elijah Smith


San Bernardino County, Population Dot Density by Block Group, 2004
Using Zones of Inclusion
Density by Block Group
1 Dot = 1,000
POP2004
1:1,700,000
1:1,100,000
N
0
10
20
40
60
80
Miles
Projection: WGS, Datum: NAD 83, Data: ESRI StreetMap Pro 2007, Authors: Alex Quintero, Daneil Elijah Smith

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