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The earth and earth coordinates

Of all the jobs maps do for you, one stands out. They tell you where things are and let you communicate this information efficiently to others. This, more than any other factor, accounts for the widespread use of maps. Maps give you a superb **locational reference system**—a way of pinpointing the position of things in space.

There are many ways to determine the location of a feature shown on a map. All of these begin with defining a geometrical figure that approximates the true shape and size of the earth. This figure is either a **sphere** or an **oblate ellipsoid** (slightly flattened sphere) of precisely known dimensions. Once the dimensions of the sphere or ellipsoid are defined, a **graticule** of east–west lines called **parallels** and north–south lines called **meridians** is draped over the sphere or ellipsoid. The angular distance of a parallel from the equator and a meridian from what we call the **prime meridian** gives us the **latitude** and **longitude** coordinates of a feature. The locations of elevations measured relative to an average gravity or sea-level surface called the **geoid** can then be defined by three-dimensional coordinates.

THE EARTH AS A SPHERE

We have known for over 2,000 years that the earth is spherical in shape. We owe this idea to several ancient Greek philosophers, particularly **Aristotle** (fourth century BC), who believed that the earth's sphericity could be proven by careful visual observation. Aristotle noticed that as he moved north or south the stars were not stationary—new stars appeared in the northern horizon while familiar stars disappeared to the south. He reasoned that this could occur only if the earth were curved north to south. He also observed that departing sailing ships, regardless of their direction of travel, always disappeared from view hull first. If the earth were flat, the ships would simply get smaller as they sailed away. Only on a sphere would hulls always disappear first. His third observation was that a circular shadow is always cast by the earth on the moon during a lunar eclipse, something that would occur only if the earth were spherical. These arguments entered the Greek literature and persuaded scholars over the succeeding centuries that the earth must be spherical in shape.

Determining the size of our spherical earth was a daunting task for our ancestors. The Greek scholar **Eratosthenes**, head of the then-famous library and museum in Alexandria, Egypt, around 250 BC, made the first scientifically based estimate of the earth's

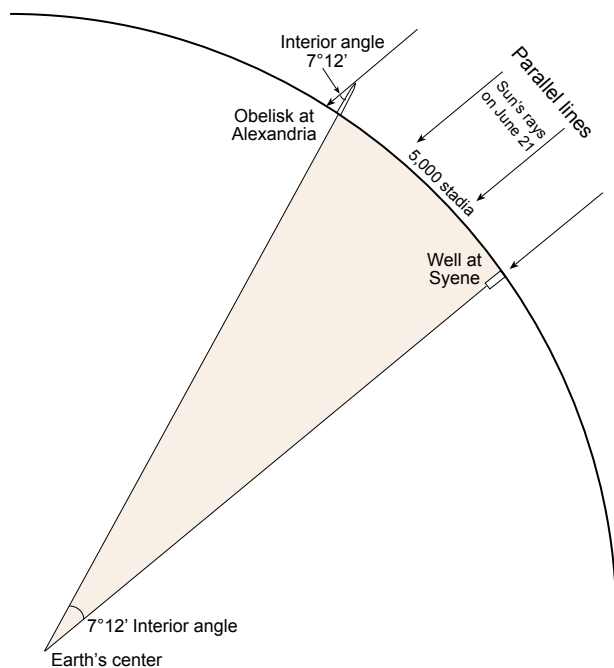


Figure 1.1 Eratosthenes's method for measuring the earth's circumference.

circumference. The story that has come down to us is of Eratosthenes reading an account of a deep well at Syene near modern Aswan about 500 miles (800 kilometers) south of Alexandria. The well's bottom was illuminated by the sun only on June 21, the day of the summer solstice. He concluded that the sun must be directly overhead on this day, with rays perpendicular to the level ground (figure 1.1). Then he reasoned brilliantly that if the sun's rays were parallel and the earth was spherical, a vertical column like an obelisk should cast a shadow in Alexandria on the same day. Knowing the angle of the shadow would allow the earth's circumference to be measured if the north–south distance to Syene could be determined. The simple geometry involved here is “if two parallel lines are intersected by a third line, the alternate interior angles are equal.” From this he reasoned that the shadow angle at Alexandria equaled the angular difference at the earth's center between the two places.

The story continues that on the next summer solstice Eratosthenes measured the shadow angle from an obelisk in Alexandria, finding it to be $7^{\circ}12'$, or $\frac{1}{50}$ th of a circle. Hence, the distance between Alexandria and Syene is $\frac{1}{50}$ th of the earth's circumference. He was told that Syene must be about 5,000 stadia south of Alexandria, since camel caravans traveling at 100 stadia per day took 50 days to make the trip between the two cities. From this distance estimate, he computed the earth's circumference as $50 \times 5,000$ stadia, or 250,000 stadia. In Greek times a **stadion** varied from 200 to 210 modern yards (182 to 192 meters), so his computed circumference was somewhere between 28,400 and 29,800 modern statute miles (45,700 and 47,960 kilometers), 14 to 19 percent greater than the current value of 24,874 statute miles (40,030 kilometers).

We now know that the error was caused by overestimation of the distance between Alexandria and Syene, and to the fact that they are not exactly north–south of each other. However, his method is sound mathematically and was the best circumference measurement until the 1600s. Equally important, Eratosthenes had the idea that careful observations of the sun would allow him to determine angular differences between places on earth, an idea that we shall see was expanded to other stars and recently to the Global Positioning System (GPS), a “constellation” of 24 earth-orbiting satellites that make it possible for people to pinpoint geographic location and elevation with a high degree of accuracy using ground receivers (see chapter 14 for more on GPS).

THE GRATICULE

Once the shape and size of the earth were known, mapmakers required some system for defining locations on the surface. We are again indebted to ancient Greek scholars for devising a systematic way of placing reference lines on the spherical earth.

Parallels and meridians

Astronomers before Eratosthenes had placed on maps horizontal lines marking the **equator** (forming the circle around the earth that is equidistant from the north and south poles) and the **tropics of Cancer and Capricorn** (marking the northernmost and southernmost positions where the sun is directly overhead on the summer and winter solstices, such as Syene). Later the astronomer and mathematician **Hipparchus** (190–125 BC) proposed that a set of equally spaced east–west lines called **parallels** be drawn on maps (figure 1.2). To these he added a set of north–south lines called **meridians** that are equally spaced at the equator and converge at the north and south poles. We now call this arrangement of parallels and meridians the **graticule**. Hipparchus’s numbering system for parallels and meridians was and still is called **latitude** and **longitude**.

Latitude and longitude

Latitude on the spherical earth is the north–south angular distance from the equator to the place of interest (figure 1.3). The numerical range of latitude is from 0° at

the equator to 90° at the poles. The letters N and S, such as 45°N for Fossil, Oregon, are used to indicate north and south latitude. **Longitude** is the angle, measured along the equator, between the intersection of the reference meridian, called the **prime meridian**, and the point where the meridian for the feature of interest intersects the equator. The numerical range of longitude is from 0° to 180° east and west of the prime meridian, twice as long as parallels. East and west longitudes are labeled E and W, so Fossil, Oregon, has a longitude of 120°W.

Putting latitude and longitude together into what is called a **geographic coordinate** (45°N, 120°W) pinpoints a place on the earth’s surface. There are several ways to write latitude and longitude values. The oldest is the **Babylonian sexagesimal system** of degrees (°), minutes (′), and seconds (″), where there are 60 minutes in a degree and 60 seconds in a minute. The latitude and longitude of the capitol dome in Salem, Oregon, is 44°56′18″N, 123°01′47″W, for example.

Sometimes you will see longitude west of the prime meridian and latitude south of the equator designated with a negative sign instead of the letters W and S. Latitude and longitude locations can also be expressed in **decimal degrees** through the following equation:

$$\text{Decimal degrees} = dd + mm / 60 + ss / 3600$$

where *dd* is the number of whole degrees, *mm* is the number of minutes, and *ss* is the number of seconds. For example:

$$44^{\circ}56'18'' = 44 + 56 / 60 + 18 / 3600 = 44.9381^{\circ}$$

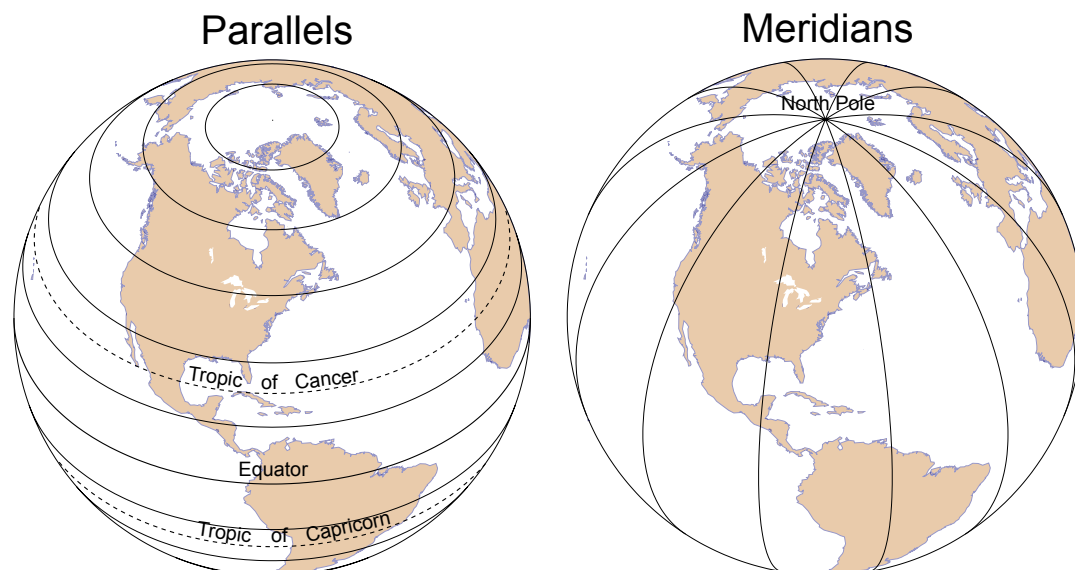


Figure 1.2 Parallels and meridians.

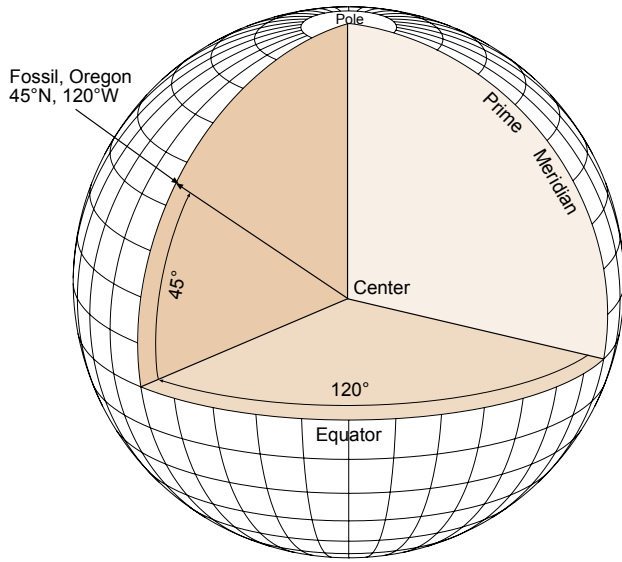


Figure 1.3 Latitude and longitude on the sphere allow the positions of features to be explicitly identified.

Decimal degrees are often rounded to two decimal places, so that the location of the Oregon state capitol dome would be written in decimal degrees as 44.94, -123.03. If we can accurately define a location to the nearest 1 second of latitude and longitude, we have specified its location to within 100 feet (30 meters) of its true location on the earth.

Prime meridians

The choice of **prime meridian** (the meridian at 0° used as the reference from which longitude east and west is measured) is entirely arbitrary because there is no physically definable starting point like the equator. In the fourth century BC Eratosthenes selected Alexandria as the starting meridian for longitude, and in medieval times the Canary Islands off the coast of Africa were used since they were then the westernmost outpost of western civilization. In the eighteenth and nineteenth centuries, many countries used their capital city as the prime meridian for the nation's maps, including the meridian through the center of the White House in Washington, D.C., for early nineteenth-century maps (see table 1.1 for a listing of historical prime meridians). You can imagine the confusion that must have existed when trying to locate places on maps from several countries. The problem was eliminated in 1884 when the **International Meridian Conference** selected as the international standard the British prime meridian—defined by the north-south optical axis of a telescope at the Royal Observatory

in Greenwich, a suburb of London. This is called the **Greenwich meridian**.

You may occasionally come across a historical map using one of the prime meridians in table 1.1, at which time knowing the angular difference between the prime meridian used on the map and the Greenwich meridian becomes very useful. As an example, you might see in an old Turkish atlas that the longitude of Seattle, Washington, is 151°16' W (based on the Istanbul meridian), and you know that the Greenwich longitude of Seattle is 122°17' W. You can determine the Greenwich longitude of Istanbul through subtraction:

$$151^{\circ}16' - 122^{\circ}17' = 28^{\circ}59'E$$

The computation is done more easily in decimal degrees as described earlier:

$$151.27 - 122.28 = 28.99 \text{ degrees}$$

Amsterdam, Netherlands	4°53'01"E
Athens, Greece	23°42'59"E
Beijing, China	116°28'10"E
Berlin, Germany	13°23'55"E
Bern, Switzerland	7°26'22"E
Brussels, Belgium	4°22'06"E
Copenhagen, Denmark	12°34'40"E
Ferro, Canary Islands	17°40'00"W
Helsinki, Finland	24°57'17"E
Istanbul, Turkey	28°58'50"E
Jakarta, Indonesia	106°48'28"E
Lisbon, Portugal	9°07'55"W
Madrid, Spain	3°41'15"W
Moscow, Russia	37°34'15"E
Oslo, Norway	10°43'23"E
Paris, France	2°20'14"E
Rio de Janeiro, Brazil	43°10'21"W
Rome (Monte Mario), Italy	12°27'08"E
St. Petersburg, Russia	30°18'59"E
Stockholm, Sweden	18°03'30"E
Tokyo, Japan	139°44'41"E
Washington, D.C., USA	77°02'14"W

Table 1.1 Prime meridians used previously on foreign maps, along with longitudinal distances from the Greenwich meridian.

THE EARTH AS AN OBLATE ELLIPSOID

Scholars assumed that the earth was a perfect sphere until the 1660s when Sir Isaac Newton developed the theory of gravity. Newton thought that mutual gravitation should produce a perfectly spherical earth if it were not rotating about its polar axis. The earth's 24-hour rotation, however, introduces outward centrifugal forces perpendicular to the axis of rotation. The amount of force varies from zero at each pole to a maximum at the equator, obeying the following equation:

$$\text{centrifugal force} = \text{mass} \times \text{velocity}^2 \times \text{distance from the axis of rotation}$$

To understand this, imagine a very small circular disk the diameter of a dinner plate (about 10 inches or 25 centimeters) at the pole and two very thin horizontal cylindrical columns, one from the center of the earth to the equator (along the **equatorial axis**) and the second perpendicular from the **polar axis** (the axis from the center of the earth to the poles) to the 60th parallel (figure 1.4). The disk at the pole has a tiny mass, but both the velocity and distance from the axis are very small, so the **centrifugal force** (the apparent force caused by the inertia of the body that draws a rotating body away from the center of rotation) is nearly 0. The column to the equator is the earth's radius in length, and the velocity increases

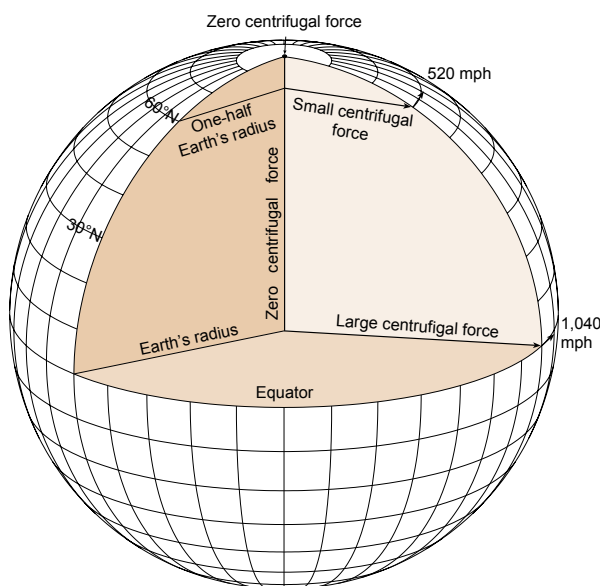


Figure 1.4 A systematic increase in centrifugal force from the pole to equator causes the earth to be an oblate ellipsoid.

from 0 at the center of the earth to a maximum of around 1,040 miles per hour (1,650 kilometers per hour) at the equator. This means that the total centrifugal force on the column is quite large. From the pole to the equator there will be a steady increase in centrifugal force. We see this at the 60th parallel where the column would be half the earth's radius in length and the velocity would be half that at the equator.

Newton noted that these outward centrifugal forces counteract the inward pull of gravity, so the net inward force decreases progressively from the pole to the equator. The column from the center of the earth to the equator extended outward slightly because of the increased centrifugal force and decreased inward gravitational force. A similar column from the center of the earth to the pole experiences zero centrifugal force and hence does not have this slight extension. Slicing the earth in half from pole to pole would then reveal an ellipse with a slightly shorter polar radius and slightly longer equatorial radius; we call these radii the **semiminor** and **semimajor axes**, respectively (figure 1.5). If we rotate this ellipse 180 degrees about its polar axis, we obtain a three-dimensional solid that we call an **oblate ellipsoid**. In the 1730s, scientific expeditions to Ecuador and Finland measured the length of a degree of latitude at the equator and near the Arctic

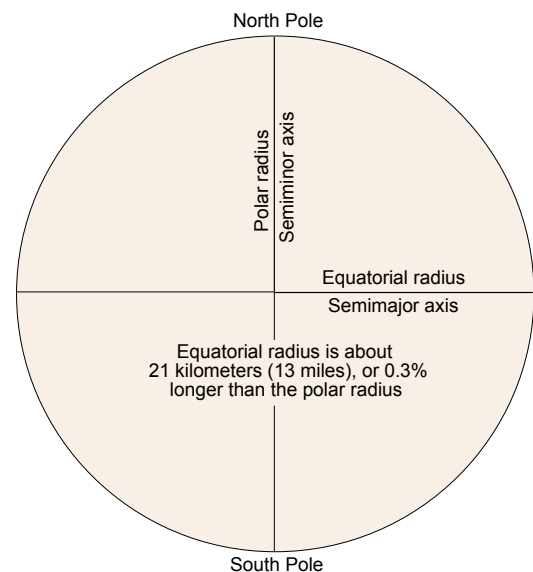


Figure 1.5 The form of the oblate ellipsoid was determined by measurements of degrees at different latitudes beginning in the 1730s. Its equatorial radius was about 13 miles (21 kilometers) longer than its polar radius. The north-south slice through the earth's center in this figure is true to scale, but our eye cannot see the deformation because it is so minimal.

Name	Date	Equatorial radius (km)	Polar radius (km)	Areas of use
WGS 84	1984	6,378.137	6,356.75231	Worldwide
GRS 80	1980	6,378.137	6,356.7523	Worldwide (NAD 83)
Australian	1965	6,378.160	6,356.7747	Australia
Krasovsky	1940	6,378.245	6,356.863	Soviet Union
International	1924	6,378.388	6,356.9119	Remainder of world not covered by older ellipsoids (European Datum 1950)
Clarke	1880	6,378.2491	6,356.5149	France; most of Africa
Clarke	1866	6,378.2064	6,356.5838	North America (NAD 27)
Bessel	1841	6,377.3972	6,356.079	Central Europe; Sweden; Chile; Switzerland; Indonesia
Airy	1830	6,377.5634	6,356.2569	Great Britain; Ireland
Everest	1830	6,377.2763	6,356.0754	India and the rest of South Asia

Table 1.2 Historical and current oblate ellipsoids.

Circle, proving Newton correct. These and additional meridian-length measurements in following decades for other parts of the world allowed the semimajor and semiminor axes of the oblate ellipsoid to be computed by the early 1800s, giving about a 13-mile (21-kilometer) difference between the two, only one-third of one percent.

Different ellipsoids

During the nineteenth century better surveying equipment was used to measure the length of a degree of latitude on different continents. From these measurements slightly different oblate ellipsoids varying by only a few hundred meters in axis length best fit the measurements. Table 1.2 is a list of these ellipsoids, along with their areas of usage. Note the changes in ellipsoid use over time. For example, the **Clarke 1866 ellipsoid** was the best fit for North America in the nineteenth century and hence was used as the basis for latitude and longitude on topographic and other maps produced in Canada, Mexico, and the United States from the late 1800s to about the late 1970s. By the 1980s, vastly superior surveying equipment coupled with millions of observations of satellite orbits allowed us to determine oblate ellipsoids that are excellent average fits for the entire earth. Satellite data are important because the elliptical shape of each orbit monitored at ground receiving stations mirrors the earth's shape. The most recent of these, called the **World Geodetic System of 1984 (WGS 84)**, replaced the Clarke 1866 ellipsoid in North America and is used as the basis for latitude and longitude on maps throughout the world. You'll see in table 1.2 that the WGS 84 ellipsoid has an equatorial

radius of 6,378.137 kilometers (3,963.191 miles) and a polar radius of 6,356.752 kilometers (3,949.903 miles).

The oblate ellipsoid is important to us because parallels are not spaced equally as on a sphere, but vary slightly in spacing from the pole to the equator. This is shown in figure 1.6, a cross section of a greatly flattened oblate ellipsoid. Notice that near the pole the ellipse curves less than near the equator. We say that on an oblate ellipsoid the **radius of curvature** (the measure of how curved the surface is) is greatest at the pole

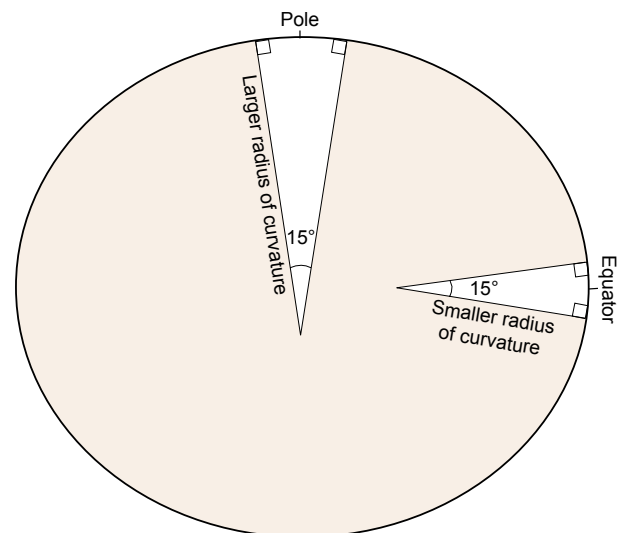


Figure 1.6 This north-south cross section through the center of a greatly flattened oblate ellipsoid shows that a larger radius of curvature at the pole results in a larger ground distance per degree of latitude relative to the equator.

and smallest at the equator. The north–south distance between two points on the surface equals the radius of curvature times the angular difference between them. For example, the distance between two points one degree apart in latitude between 0° and 1° at the equator is 68.703 miles (110.567 kilometers), shorter than the 69.407 miles (111.699 kilometers) between two points at 89° and 90° north latitude. This example shows that the spacing of parallels decreases slightly from the pole to the equator.

Geodetic latitude

Geodetic latitude is defined as the angle made by the horizontal equator line and a line perpendicular to the ellipsoidal surface at the parallel of interest (figure 1.7). Geodetic latitude differs from latitude on a sphere because of the unequal spacing of parallels on the ellipsoid. Lines perpendicular to the ellipsoidal surface only pass through the center of the earth at the poles and equator, but all lines perpendicular to the surface of a sphere will pass through its center. This is why the latitude defined by these lines on a sphere is called **geocentric latitude**.

Defining geodetic latitude in this way means that geocentric and geodetic latitude are identical only at 0° and 90°. Everywhere else geocentric latitude is slightly smaller than the corresponding geodetic latitude, as shown in table 1.3. Notice that the difference between geodetic and geocentric latitude increases in a symmetrical fashion from zero at the poles and equator to a maximum of just under one-eighth of a degree at 45°.

Geocentric	Geodetic	Geocentric	Geodetic
0°	0.000°	50°	50.126°
5	5.022	55	55.120
10	10.044	60	60.111
15	15.064	65	65.098
20	20.083	70	70.082
25	25.098	75	75.064
30	30.111	80	80.044
35	35.121	85	85.022
40	40.126	90°	90.000°
45°	45.128°		

Table 1.3 Geocentric and corresponding geodetic latitudes (WGS 84) at 5° increments.

Geodetic longitude

It turns out that there is no need to make any distinction between geodetic longitude and geocentric longitude. While the definition of geocentric longitude is mathematically different from geodetic longitude, the end result is essentially the same. The angle between the line from a point on the surface of the earth to the center of the earth and then to the prime meridian determines the longitude. Because of the nature of the ellipsoidal model, this turns out to be the same as the angle for geocentric longitude, which was described above.

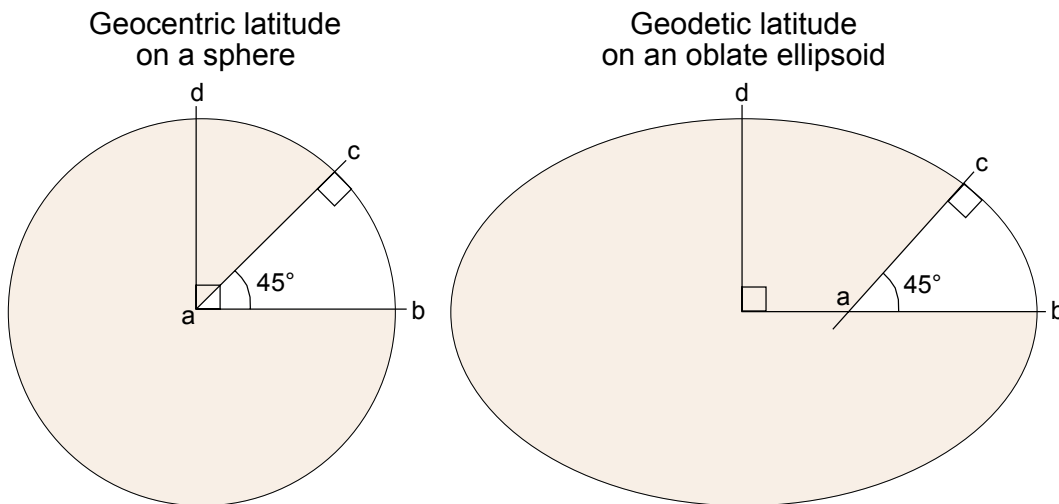


Figure 1.7 Geocentric and geodetic latitudes of 45°. On a sphere, circular arc distance b–c is the same as circular arc distance c–d. On the greatly flattened oblate ellipsoid, elliptical arc distance b–c is less than elliptical arc distance c–d. On the WGS 84 oblate ellipsoid, arc distance b–c is 3,097.50 miles (4,984.94 kilometers) and arc distance c–d is 3,117.43 miles (5,017.02 kilometers), a difference of about 20 miles (32 kilometers).

DETERMINING GEODETIC LATITUDE AND LONGITUDE

The oldest way to determine geodetic latitude and longitude is with instruments for observing the positions of celestial bodies. The essence of the technique is to establish celestial lines of position (east–west, north–south) by comparing the predicted positions of celestial bodies with their observed positions. A handheld instrument, called a **sextant**, historically was the tool used to measure the angle (or altitude) of a celestial body above the earth’s horizon (figure 1.8). Before GPS, it was the tool that nautical navigators used to find their way using the moon, planets, and stars, including our sun.

Astronomers study and tabulate information on the actual motion of celestial bodies that helps to pinpoint latitude and longitude. Since the earth rotates on an axis defined by the north and south poles, stars in the northern hemisphere’s night sky appear to move slowly in a circle centered on **Polaris** (the **North Star**). The navigator needs only to locate Polaris to find north. In addition, because the star is so far away from the earth, the angle from the horizon to Polaris is the same as the latitude (figure 1.9).

In the southern hemisphere, latitude is harder to determine because four stars are used to interpolate due south. Because there is no equivalent to Polaris over the south pole, navigators instead use a small constellation called **Crux Australis** (the **Southern Cross**) to serve the same function (figure 1.10). Finding south is more complicated because the Southern Cross is a collection of five stars that are part of the constellation Centaurus. The four outer stars form a cross, while the fifth much

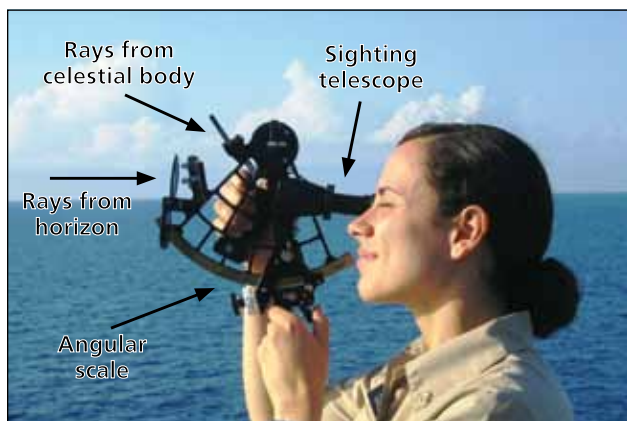
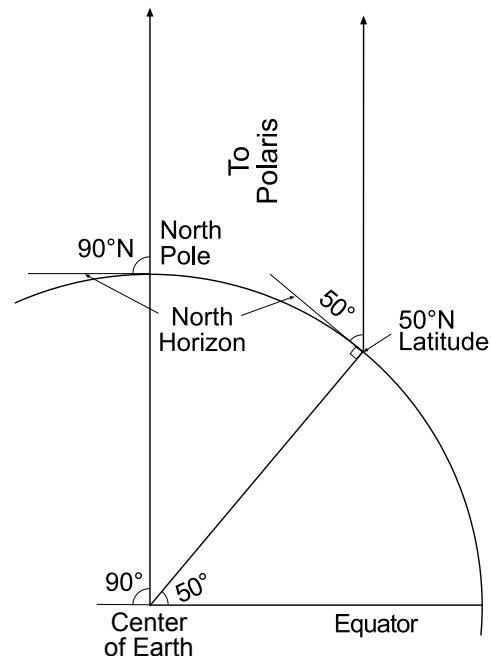


Figure 1.8 A sextant is used at sea to find latitude from the vertical angle between the horizon and a celestial body such as the sun.

dimmer star (Epsilon) is offset about 30 degrees below the center of the cross.

Geodetic longitude can be determined in a straightforward manner. As we saw earlier, the prime meridian at 0° longitude passes through Greenwich, England. Therefore, each hour difference between your time and that at Greenwich, called **Greenwich mean time** or **GMT**, is roughly equivalent to 15° of longitude from Greenwich. To determine geodetic longitude, compare your local time with Greenwich mean time and multiply by 15° of longitude for each hour of difference. The difficulty is that time is conventionally defined by broad time zones, not by the exact local time at your longitude. Your exact local time must be determined by celestial observations.

In previous centuries, accurately determining longitude was a major problem in both sea navigation and mapmaking. It was not until 1762 that a clock portable enough to take aboard ship and accurate enough for longitude finding was invented by the Englishman John Harrison. This clock, called a **chronometer**, was set to Greenwich mean time before departing on a long voyage. The longitude of a distant locale was found by noting the Greenwich mean time at local noon (the highest point of the sun in the sky, found with a sextant). The time difference was simply multiplied by 15 to find the longitude.



Courtesy of Dr. Bernie Bernard.

Figure 1.9 In the northern hemisphere, it is easy to determine your latitude by observing the height of Polaris above your northern horizon.

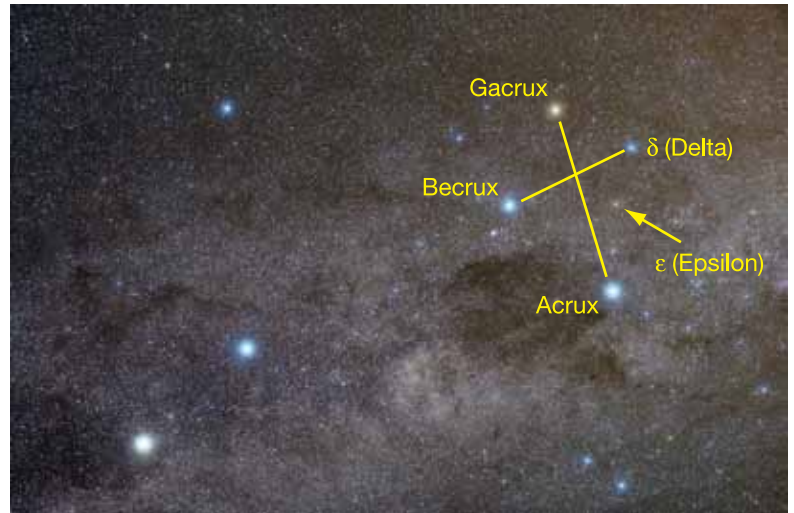


Figure 1.10 The Southern Cross is used for navigation in the southern hemisphere.

Courtesy of Dr. Yuri Beletsky.

PROPERTIES OF THE GRATICULE

Circumference of the authalic and other spheres

When determining latitude and longitude we sometimes use a different approximation to the earth than the oblate ellipsoid. Using a sphere leads to simpler calculations, especially when working with small-scale maps of countries, continents, or the entire earth (see the next chapter for more on small-scale maps). On these maps differences between locations on the sphere and the ellipsoid are negligible. The value of the earth's spherical circumference used in this book is for what cartographers call an **authalic sphere**. The authalic (meaning “area-preserving”) sphere is a sphere with the same surface area as a reference ellipsoid we are using. The equatorial and polar radii of the WGS 84 ellipsoid are what we used to calculate the radius and circumference of the authalic sphere that is equal to the surface area of the WGS 84 ellipsoid. The computations involved are moderately complex and best left to a short computer program, but the result is a sphere with a radius of 3,958.76 miles (6,371.017 kilometers) and circumference 24,873.62 miles (40,030.22 kilometers).

There are other properties of an oblate ellipsoid that we can use to define the circumference of a spherical earth. A **rectifying sphere**, for example, is one where the length of meridians from equator to pole on the ellipsoid equals one-quarter of the spherical circumference. For the WGS 84 ellipsoid, the rectifying sphere is of radius 3,956.55 miles (6,367.449 kilometers) and circumference 24,859.73 miles (40,007.86 kilometers). We can also use the 3,964.038 mile (6,378.137 kilometer) equatorial radius of the WGS 84 ellipsoid, giving a sphere of circumference 24,901.46 miles (40,075.017 kilometers).

This sphere is a little over three-tenths of a percent larger in surface area than the WGS 84 ellipsoid. These circumference values are all very close to each other, differing by less than two-tenths of a percent.

Spacing of parallels

As we saw earlier, on a spherical earth the north–south ground distance between equal increments of latitude does not vary. However, it is important to know how you want to define the distance. As we will see below, there are different definitions for terms you may take for granted, such as a “mile.”

Using the WGS 84 authalic sphere circumference, latitude spacing is always $24,874 \text{ miles} \div 360^\circ$ or 69.09 statute miles per degree. Expressed in metric and nautical units, it is 111.20 kilometers and 60.04 nautical miles per degree of latitude (see table C.1 in appendix C for the metric and English distance equivalents used to arrive at these values.)

You can see that there is about a 15 percent difference in the number of statute and nautical miles per degree. **Statute miles** are what we use for land distances in the United States, while **nautical miles** are used around the world for maritime and aviation purposes. A statute mile is about 1,609 meters, while the international standard for a nautical mile is 1,852 meters exactly (about 1.15 statute miles). The original nautical mile was defined as 1 minute of latitude measured north–south along a meridian. Kilometers are also closely tied to distances along meridians, since the meter was initially defined as one ten-millionth of the distance along a meridian from the equator to the north or south pole.

We have seen that parallels on an oblate ellipsoid are not spaced equally, as they are on a sphere, but decrease slightly from the pole to the equator. You can see in

table C.2 (in appendix C) the variation in the length of a degree of latitude for the WGS 84 ellipsoid, measured along a meridian at one-degree increments from the equator to the pole. You can see that the distance per degree of latitude ranges from 69.407 statute miles (111.699 kilometers) at the pole to 68.703 statute miles (110.567 kilometers) at the equator. The graph in figure 1.11 shows how these distances differ from the constant values of 69.09 and 69.07 statute miles (111.20 and 111.16 kilometers) per degree for the authalic and rectifying spheres, respectively. For both spheres, the WGS 84 ellipsoid distances per degree are about 0.3 miles (0.48 kilometers) greater than the sphere at the pole and 0.4 miles (0.64 kilometers) less at the equator. The ellipsoidal and spherical distances are almost the same in the mid-latitudes, somewhere between 45 and 50 degrees.

Converging meridians

A quick glance at any world globe (or figure 1.2) shows that the length of a degree of longitude, measured east-west along parallels, decreases from the equator to the pole. The precise spacing of meridians at a given latitude is found by using the following equation:

$$69.09 \text{ miles (or 111.20 kilometers)} / \text{deg.} \times \cos(\text{latitude}).^*$$

At 45° north or south of the equator, for example, $\cos(45^\circ) = 0.7071$. Therefore, the length of a degree of longitude is

$$69.09 \times 0.7071, \text{ or } 48.85 \text{ statute miles} \\ (111.20 \times 0.7071 \text{ or } 78.63 \text{ kilometers})$$

This is roughly 20 miles (32 kilometers) shorter than the 69.09-mile (111.20-kilometer) spacing at the equator.

Great and small circles

A **great circle** is the largest possible circle that could be drawn on the surface of the spherical earth. Its circumference is that of the sphere, and its center is the center of the earth so that all great circles divide the earth into halves. Notice in figure 1.3 that the equator is a great circle dividing the earth into northern and southern hemispheres. Similarly, the prime meridian and the 180° meridian at the opposite side of the earth (called the **antipodal meridian**) form a great circle dividing the earth into eastern and western hemispheres. All other pairs of meridians and their antipodal meridians are also

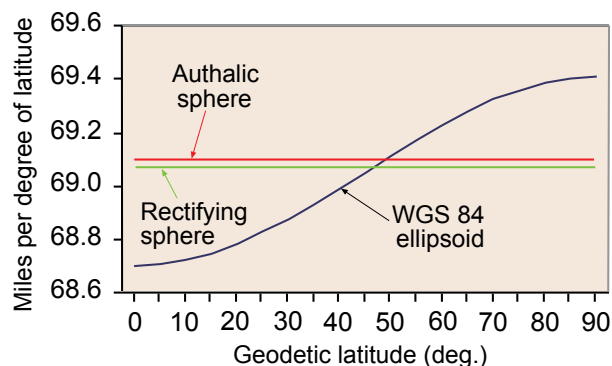


Figure 1.11 Distances along the meridian for one-degree increments of latitude from the equator to the pole on the WGS 84 ellipsoid, authalic sphere, and rectifying sphere.

great circles. A great circle is the shortest route between any two points on the earth and hence great circle routes are fundamental to long-distance navigation.

Any circle on the earth's surface that intersects the interior of the sphere at any location other than the center is called a **small circle**, and its circumference is smaller than a great circle. You can see in figure 1.3 that all parallels other than the equator are small circles. The circumference of a particular parallel is given by the following equation:

$$24,874 \text{ miles (or 40,030 kilometers)} \times \cos(\text{latitude})$$

For example, the circumference of the 45th parallel is

$$24,874 \times 0.7071 \text{ or } 17,588 \text{ statute miles} \\ (40,030 \times 0.7071 \text{ or } 28,305 \text{ kilometers})$$

Quadrilaterals

Quadrilaterals are areas bounded by equal increments of latitude and longitude, 10° by 10° in figure 1.3, for example. Since meridians converge toward the poles, the shapes of quadrilaterals vary from a square on the sphere at the equator to a very narrow spherical triangle at the pole. The equation $\cos(\text{latitude})$ gives the **aspect ratio** (width/height) of any quadrilateral. A quadrilateral centered at 45°N will have an aspect ratio of 0.7071, whereas a quadrilateral at 60°N will have an aspect ratio of 0.5. A map covering 10° by 10° will look very long and narrow at this latitude.

* Inexpensive engineering calculators can be used to compute trigonometric functions such as cosine. You can also find trig function calculators on the Internet.

GRATICULE APPEARANCE ON MAPS

Small-scale maps

World or continental maps such as globes and world atlas sheets normally use coordinates based on an authalic sphere. There are several reasons for this. Prior to using digital computers to make these types of maps numerically, it was much easier to construct them from spherical coordinates. Equally important, the differences in the plotted positions of spherical and corresponding geodetic parallels become negligible on maps that cover so much area.

You can see this by looking again at table 1.3. Earlier we saw that the maximum difference between spherical and geodetic latitude is 0.128° at the 45th parallel. Imagine drawing parallels at 45° and 45.128° on a map scaled at one inch per degree of latitude. The two parallels will be drawn a very noticeable 0.128 inches (0.325 centimeters) apart. Now imagine drawing the parallels on a map scaled at one inch per 10 degrees of latitude. The two parallels will now be drawn 0.013 inches (0.033 centimeters) apart, a difference that would not even be noticeable given the width of a line on a piece of paper. This scale corresponds to a world wall map approximately 18 inches (46 centimeters) high and 36 inches (92 centimeters) wide.

Large-scale maps

Parallels and meridians are shown in different ways on different types of maps. Topographic maps in the United States and other countries have tick marks showing the location of the graticule. All U.S. Geological Survey 7.5-minute topographic maps, for example, have graticule ticks at 2.5-minute intervals of latitude and longitude (figure 1.12). The full latitude and longitude is printed in each corner, but only the minutes and seconds of the intermediate edge ticks are shown. Note the four “+” symbols used for the interior 2.5-minute graticule ticks.

The graticule is shown in a different way on nautical charts (figure 1.13). Alternating white and dark bars spaced at the same increment of latitude and longitude line the edge of the chart. Because of the convergence of meridians, the vertical bars on the left and right edges of the chart showing equal increments of latitude are longer than the horizontal bars at the top and bottom. Notice the more closely spaced ticks beside each bar, placed every tenth of a minute on the chart in figure 1.13. These ticks are used to more precisely find the latitude and longitude of mapped features.

Aeronautical charts display the graticule another way. The chart segment for a portion of the Aleutian Islands in Alaska (figure 1.14) shows that parallels and meridians are drawn at 30-minute latitude and longitude intervals. Ticks are placed at 1-minute increments along each graticule line, allowing features to be located easily to within a fraction of a minute.

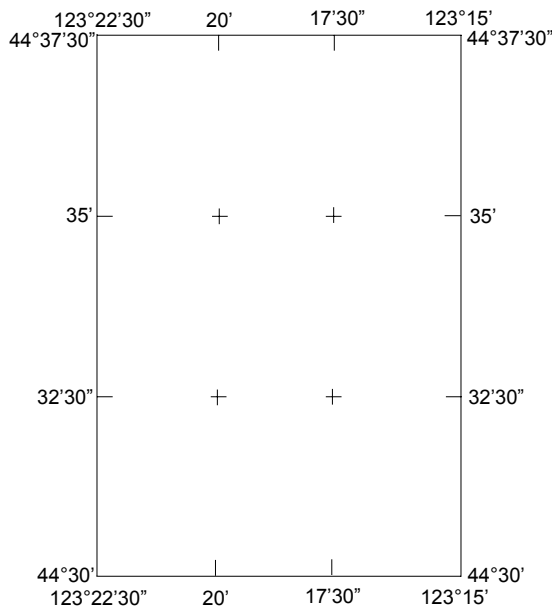
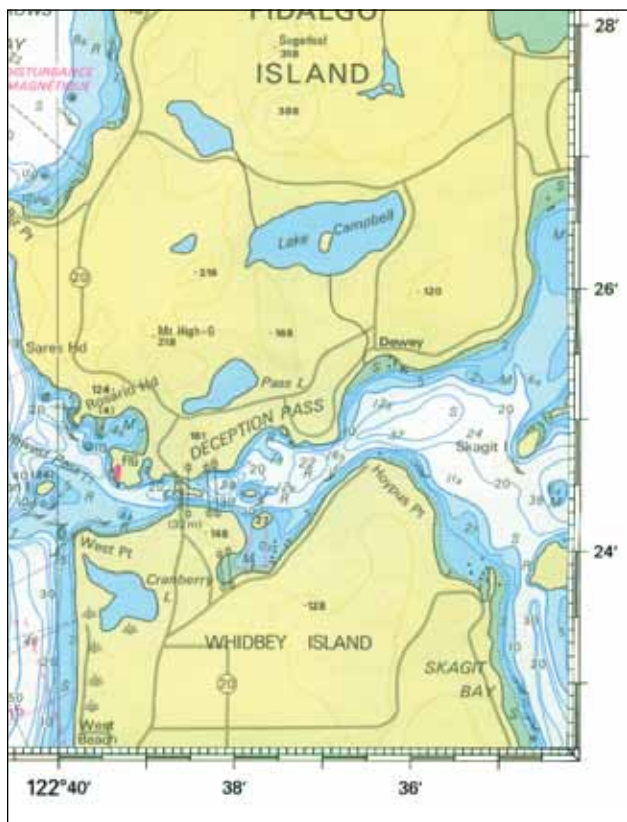


Figure 1.12 Graticule ticks on the Corvallis, Oregon, 7.5-minute topographic map.

GEODETIC LATITUDE AND LONGITUDE ON LARGE-SCALE MAPS

You will always find parallels and meridians of geodetic latitude and longitude on detailed maps of smaller extents. This is done to make the map a very close approximation to the size and shape of the piece of the ellipsoidal earth that it represents. To see the perils of not doing this, you only need to examine one-degree quadrilaterals at the equator and pole, one ranging from 0° to 1° and the second from 89° to 90° in latitude.

You can see in table C.2 (in appendix C) that the ground distance between these pairs of parallels on the WGS 84 ellipsoid is 68.703 and 69.407 miles (110.567 and 111.699 kilometers), respectively. If the equatorial quadrilateral is mapped at a scale such that it is 100 inches (254 centimeters) high, the polar quadrilateral mapped



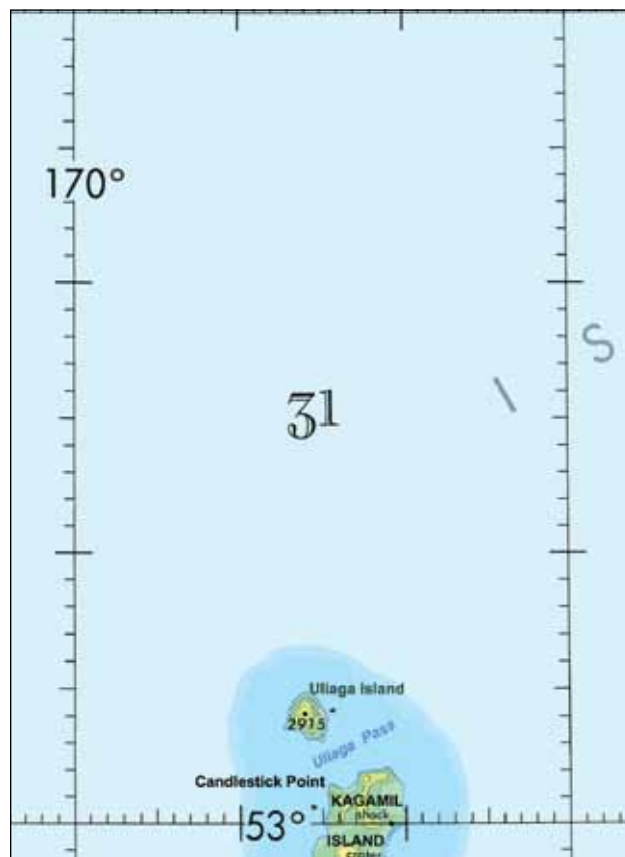
Reproduced with the permission of the Canadian Hydrographic Service.

Figure 1.13 Graticule bars on the edges of a nautical chart segment.

at the same scale will be 101 inches (256.5 centimeters) long. If we mapped both quadrilaterals using the authalic sphere having 69.09 miles (111.20 kilometers) per degree, both maps would be 100.6 inches (255.5 centimeters) long. Having both maps several tenths of an inch (or around a centimeter) longer or shorter than they should really be is an unacceptably large error for maps used to make accurate measurements of distance, direction, or area.

Horizontal reference datums

To further understand the use of different types of coordinates on detailed maps of smaller extents, we must first look at datums—the collection of very accurate **control points** (points of known accuracy) surveyors use to georeference all other map data (see chapter 5 for more on control points and georeferencing). Surveyors determine the precise geodetic latitude and longitude of horizontal control points spread across the landscape. You may have seen a horizontal control point **monument** (a fixed object established



Courtesy of the National Aeronautical Charting Office.

Figure 1.14 Graticule ticks on a small segment of an aeronautical chart.

by surveyors when they determined the exact location of a point) like figure 1.15 on the ground on top of a hill or other prominent feature. From the 1920s to the early 1980s these control points were surveyed relative to the surface of the Clarke 1866 ellipsoid, together forming what was called the **North American Datum of 1927 (NAD 27)**. Topographic maps, nautical and aeronautical charts, and many other large-scale maps of this time period had graticule lines or ticks based on this datum. For example, the southeast corner of the Corvallis, Oregon, topographic map first published in 1969 (figure 1.16) has an NAD 27 latitude and longitude of 44°30'N, 123°15'W.

By the early 1980s, better knowledge of the earth's shape and size and far better surveying methods led to the creation of a new horizontal reference datum, the **North American Datum of 1983 (NAD 83)**. The NAD 27 control points were corrected for surveying errors where required, then these were added to thousands of more recently acquired points. The geodetic latitudes and longitudes of all these points were

THE EARTH AS A GEOID

When we treat the earth as a smooth athermal sphere or oblate ellipsoid, we neglect mountain ranges, ocean trenches, and other surface features that have vertical relief. There is justification for doing this, as the earth's surface is truly smooth when we compare the surface undulations to the 7,918-mile (12,742-kilometer) diameter of the earth based on the athermal sphere. The greatest relief variation is the approximately 12.3-mile (19.8-kilometer) difference between the summit of Mt. Everest (29,035 feet or 8,852 meters) and the deepest point in the Mariana Trench (36,192 feet or 11,034 meters). This vertical difference is immense on our human scale, but it is only $\frac{1}{640}$ th of the earth's diameter. If we look at the difference between the earth's average land height (2,755 feet or 840 meters) and ocean depth (12,450 feet or 3,795 meters), the average roughness is only $\frac{4,635\text{m}}{12,742\text{km}}$, or $\frac{1}{2,750}$ th of the diameter. It has been said that if the earth were reduced to the diameter of a bowling ball, it would be smoother than the bowling ball!

The earth's global-scale smoothness aside, knowing the elevations and depths of features is very important to us. Defining locations by their geodetic latitude, longitude, and elevation gives you a simple way to collect elevation data and display this information on maps. The top of Mt. Everest, for example, is located at $27^{\circ}59'N$, $86^{\circ}56'E$, 29,035 feet (8,852 meters), but what is this elevation relative to? This leads us to another approximation of the earth called the **geoid**, which is a surface of equal gravity used as the reference for elevations.

Vertical reference datums

Elevations and depths are measured relative to what is called a **vertical reference datum**, an arbitrary surface with an elevation of zero. The traditional datum used for land elevations is **mean sea level (MSL)** (see chapter 6 for more on mean sea level). Surveyors define MSL as the average of all low and high tides at a particular starting location over a **metonic cycle** (the 19-year cycle of the lunar phases and days of the year). Early surveyors chose this datum because of the measurement technology of the day. Surveyors first used the method of **leveling**, where elevations are determined relative to the point where mean sea level is defined, using horizontally aligned telescopes and vertically aligned leveling rods (see chapter 6 for more on leveling). A small circular monument was placed in the ground at each surveyed benchmark elevation point. A **benchmark** is a permanent monument that establishes the exact elevation of a place.

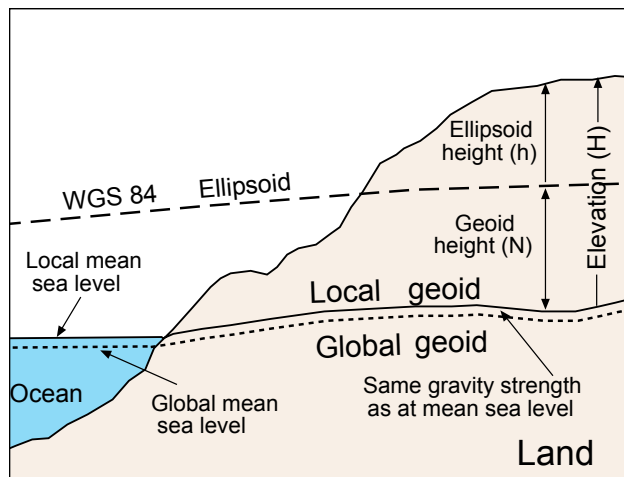


Figure 1.17 The geoid is the surface where gravity is the same as at mean sea level. Elevations traditionally have been measured relative to the geoid, but modern GPS-determined heights are relative to the WGS 84 ellipsoid.

Later, surveyors could determine elevation by making gravity measurements at different locations on the landform and relating them to the strength of gravity at the point used to define MSL. Gravity differences translate into elevation differences.

Mean sea level is easy to determine along coastlines, but what about inland locations? What is needed is to extend mean sea level across the land. Imagine that the mean sea level is extended under the continental land masses, which is the same thing as extending a surface having the same strength of gravity as mean sea level (figure 1.17). This imaginary equal gravity surface doesn't form a perfect ellipsoid, however, because differences in topography and earth density affect gravity's pull at different locations. The slightly undulating nearly ellipsoidal surface that best fits mean sea level for all the earth's oceans is called a **global geoid**. The global geoid rises and falls approximately 100 meters above and below the oblate ellipsoid surface in an irregular fashion. World maps showing land topography and ocean bathymetry use land heights and water depths relative to the global geoid surface.

The mean sea level datum based on the geoid is so convenient that it is used to determine elevations around the world and is the base for the elevation data found on nearly all topographic maps and nautical charts. But be aware that the **local geoid** used in your area is probably slightly above or below (usually within two meters) the global geoid elevations used on world maps. This difference is caused by mean sea level at one or more locations being used as the vertical reference datum for your nation or continent, not the average sea level for all the oceans.

In the United States, for example, you may see elevations relative to the **National Geodetic Vertical Datum of 1929 (NGVD 29)** on older topographic maps. This datum was defined by the observed heights of mean sea level at 26 tide gauges, 21 in the United States and 5 in Canada. It also was defined by the set of elevations of all benchmarks resulting from over 60,000 miles (96,560 kilometers) of leveling across the continent, totaling over 500,000 vertical control points. In the late 1980s, surveyors adjusted the 1929 datum with new data to create the **North American Vertical Datum of 1988 (NAVD 88)**. Topographic maps, nautical charts, and other cartographic products made from this time forward have used elevations based on NAVD 88. Mean sea level for the continent was defined at one tidal station on the St. Lawrence River at Rimouski, Quebec, Canada. NAVD 88 was a necessary update of the 1929 vertical datum since about 400,000 miles (650,000 kilometers) of leveling had been added to the NGVD since 1929. Additionally, numerous benchmarks had been lost over the decades—and the elevations at others had been affected—by vertical changes caused by rising of land elevations

since the retreat of glaciers at the end of the last ice age, or subsidence from sedimentation and the extraction of natural resources like oil and water.

GPS has created a second option for measuring elevation (see chapter 14 for more on GPS). GPS receivers calculate what is called the **ellipsoidal height** (h), the distance above or below the surface of the WGS 84 ellipsoid along a line from the surface to the center of the earth (figure 1.17). An ellipsoidal height is not an elevation, since it is not measured relative to the mean sea level datum for your local geoid. Therefore, you must convert GPS ellipsoidal height values to mean sea level datum elevations (H) before you can use them with existing maps. You do this by subtracting the geoid height (N) at each point from the ellipsoid height (h) measured by the GPS receiver using the equation $H = h - N$. The look-up table needed to make this conversion usually is stored in your GPS receiver's computer. In the conterminous United States, geoid heights range from a low of -51.6 meters in the Atlantic Ocean to a high of -7.2 meters in the Rocky Mountains (figure 1.18). Worldwide, geoid heights vary from -105 meters just south of Sri Lanka to 85 meters in Indonesia.

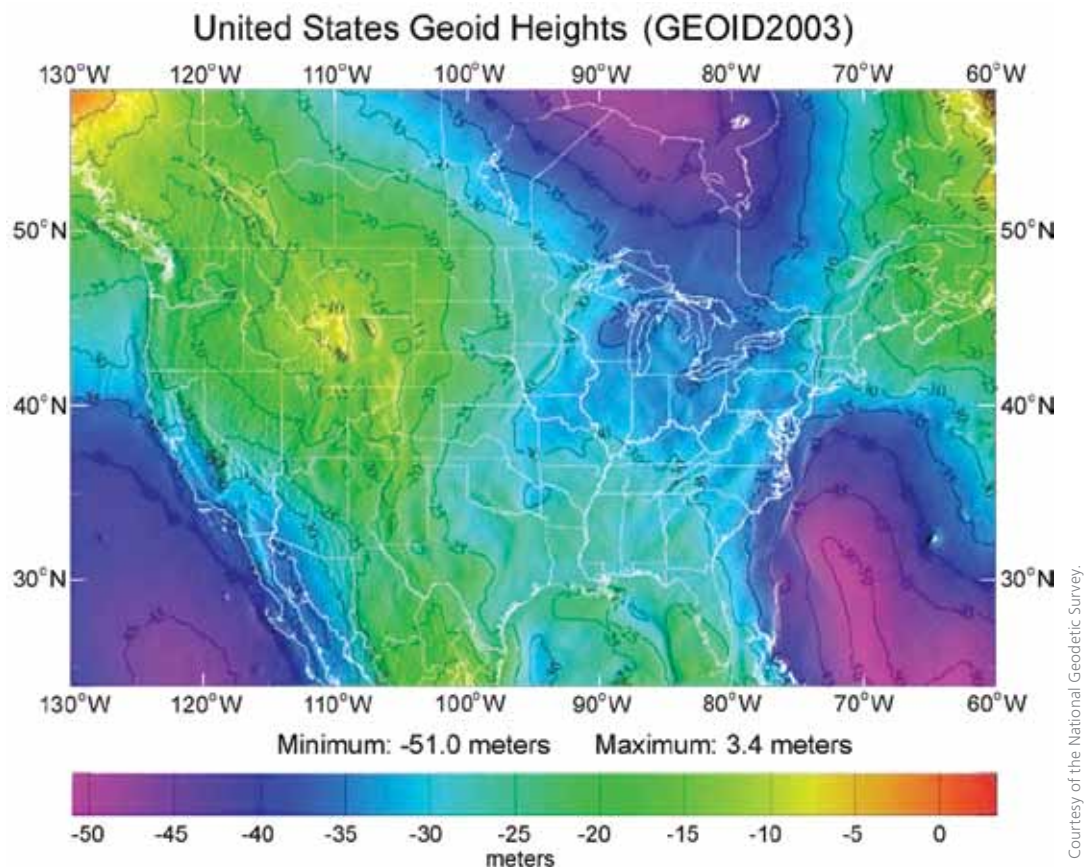


Figure 1.18 Geoid heights in the United States and vicinity (from the National Geodetic Survey GEOID2003 model).

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